UNITED STATES OF AMERICA
BEFORE THE
FEDERAL ENERGY REGULATORY COMMISSION

PJM Interconnection, L.L.C.	))))	Docket No. EL21-91-000, -003; ER21-1635-010

REPLY COMMENTS OF THE INDEPENDENT MARKET MONITOR FOR PJM

Pursuant to Rule 602(f) of the Commission's Rules and Regulations,[1] Monitoring Analytics, LLC, acting in its capacity as the Independent Market Monitor ("Market Monitor") for PJM Interconnection, L.L.C.[2] ("PJM"), submits these reply comments in support of its offer of settlement and Settlement Agreement filed September 10, 2024 ("IMM Offer").

On October 10, 2024, comments opposing the IMM Offer were filed by PJM, by the Indicated Suppliers[3] and by FERC Trial Staff ("Staff").

The IMM Offer is properly filed, serves the public interest and should be approved. The IMM Offer is supported by substantial evidence. There is no "genuine dispute about material facts" pertaining to the IMM Offer.

The IMM Offer should be approved on the merits. This case presents a straightforward issue. Should resources receiving a special formula rate tied to the recovery of investment receive, based on a known faulty input, payment for costs based on taxes that they are not required to pay. Despite the simple issue presented by this case, redress has been delayed by slow administration of the tariff revision process by PJM, and by the lengthy procedures before the Commission. A consequence of this delay is that for years generators

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1 18 CFR § 385.602(f) (2024).

2 Capitalized terms used herein and not otherwise defined have the meaning used in the PJM Open Access Transmission Tariff ("OATT").

3 The Indicated Suppliers include: Dynegy Marketing and Trade, LLC and Vistra Corp., Hazleton Generation LLC, J-POWER USA Development Co., Ltd., and LS Power Development, LLC.

have been collecting from customer revenues to pay taxes that they are not required to pay. As of November 17, 2022, refund protection in this Section 206 proceeding expired.[4]

Approving the IMM Offer would avoid an unjustified windfall to affected black start unit owners of the largest possible portion, without continued litigation, of the $89.7 million in overcharges that will result from applying incorrectly calculated formula rates.[5]

Action by the Commission is needed to ensure that the formula rate, the black start unit Capital Cost Recovery Rate, of which the CRF is a component input, operates as it was intended, allows unit owners recovery of specific investment costs but no more, sets just and reasonable rates for PJM customers, and serves the public interest. The IMM Offer avoids retroactive ratemaking.

Based on the record of this proceeding, including the Affidavit and supporting attachments included with the IMM Offer, PJM should be required to provide accurate CRF values for units selected for black start service prior to June 6, 2021. There is no reason why this proceeding cannot be immediately resolved with the just and reasonable, accurate, implementation of the formula rates for black start service in PJM, recognizing the recovery of capital to date and the remaining term of the CRF values.

I. REPLY COMMENTS

A. The IMM Offer Is Properly Filed.

While acknowledging that "the Commission's rules do not prohibit unilateral offers of settlement," Indicated Suppliers argue (at 3–5) that the IMM Offer "should be summarily rejected." Indicated Suppliers variously complain that the IMM Offer "reflects the litigation position taken by the IMM throughout these proceedings." The IMM Offer reflects the position that the Market Monitor has taken in settlement discussions and is consistent with its position in litigation because it is an approach that correctly accounts for the impact of the

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4 See PJM Interconnection, L.L.C.; Notice of Institution of Section 206 Proceeding and Refund Effective Date, Docket No. EL21-19-000, 86 Fed. Reg. 45980 (August 17, 2021); 16 U.S.C. § 824e(b).

5 *See* Bowring Affidavit at 20:20–21.

Tax Cuts and Jobs Act ("TCJA") [6] on the CRF values in paragraph 18 of Schedule 6A to the OATT that are a component of the special formula rate at issue in this case.

Contrary to the claim that the Market Monitor has not engaged in "discussions prior to filing" the IMM Offer, the IMM Offer follows the approach that the Market Monitor has been discussing since PJM initiated this proceeding on April 7, 2021, but concedes for purposes of settlement a limitation to prospective relief. The Market Monitor appreciates that the Indicated Suppliers believe that they can leverage a better outcome for themselves than a just and reasonable result and have not agreed to the IMM Offer and are contesting the IMM Offer. The Market Monitor cannot and has not imposed the IMM Offer on anyone. The IMM Offer can only take effect if the Commission agrees with the Market Monitor that the approach is just and reasonable. If the Commission agrees with the logical basis for the IMM Offer, there are no practical alternatives to the proposed approach.

The IMM Offer is not exactly the same as the Market Monitor's litigation position. The Market Monitor continues to believe that the best outcome to this proceeding would be to recalculate the formula rate using the correct CRF component and apply that formula for the entire term of each CRF. Longstanding precedent on the application of the filed rate doctrine supports the Market Monitor's litigation position.[7] The Market Monitor recognizes the statement in the hearing order that the Commission would not order recalculation of the formula rates.[8] This means that the expiration of the refund period and the end of terms of some of the Capital Cost Recovery rates at issue will result in a windfall to suppliers. Forgoing continued litigation of the issue and proposing an offer that unavoidably results in a windfall to suppliers is a concession. The IMM Offer will result in a just and reasonable

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[6] Tax Cuts and Jobs Act of 2017, Pub. L. No. 115-97, 131 Stat. 2054 (2017).

[7] Comments of the Independent Market Monitor for PJM, Docket No. ER21-1635-000 (April 28, 2021) at 3–5; Request for Rehearing of the Independent Market Monitor for PJM, Docket No. ER21-1635-001, et al. (September 9, 2021) at 3–5.

[8] *See* Bowring Affidavit at 16:29–30, citing *PJM Interconnection, L.L.C.*, 176 FERC ¶ 61,080 at P 50 (2021).

outcome defined by the parameters in the hearing order, but it will not achieve the logically correct outcome that is the Market Monitor's litigation position.

Precedent exists for consideration of the IMM Offer. In a case involving rates established under Part V of the OATT for service after a unit requested deactivation, the Market Monitor filed an offer of settlement in competition with the an offer of settlement filed by other parties.[9] The Market Monitor's settlement was not accepted, but it was considered.[10] The IMM Offer is properly filed under Rule 602 and should be considered by the Commission.

B. The IMM Offer Should Be Certified to the Commission.

Indicated Suppliers state (at 26–27) that "the Offer can only be certified under Rule 602 if the Presiding Judge 'determines that there is no genuine issue of material fact,' [n103: 18 C.F.R. § 385.602(h)(2)(ii) (2024).] or if the initial decision is waived under Rule 710(d) and the Presiding Judge 'determines that the record contains substantial evidence from which the Commission may reach a reasoned decision on the merits of the contested issues'[18 C.F.R. § 385.602(h)(2)(iii)(B) (2024)]." The Market Monitor does not dispute the standard applicable to certification. The Market Monitor discussion of *Trailblazer* identifies the just and reasonable standard for which "substantial evidence" is required. The IMM Offer satisfies the standards for certification under Rule 602 and should be forwarded to the Commission.

1. There Are No Genuine Issues of Material Fact.

The Commission set one issue of material fact for hearing:

> [W]hether, as a result of changes from the TCJA, the existing CRF values result in a Capital Cost Recovery Rate for generating units that were selected to provide Black Start Service prior to June 6, 2021 that is unjust and unreasonable. While the record does not contain conclusive evidence that the existing CRF values include a 35% tax rate, the Market Monitor has introduced sufficient

9 *See GenOn Power Midwest, LP*, 149 FERC ¶ 61,218 (2014).

10 *Id*.

> evidence that those values may include a 35% tax rate, raising a disputed issue of material fact as to whether changes to the tax rate render the existing CRF values unjust and unreasonable. The import of the tax rate in the determination of the CRF value is a material fact that cannot be determined based on the existing record, which warrants setting the justness and reasonableness of the existing CRF values for hearing and settlement judge procedures.[11]

Even though the issue was set for hearing, the Commission acknowledged that "the Market Monitor has introduced sufficient evidence that those values may include a 35% tax rate."[12]

The Market Monitor agrees with Staff's statement and analysis (at 8–9), as it applies to the IMM Offer:

> As the Commission explained in Williams Natural Gas Co., '[a] hearing is not necessary to resolve factual disputes which only concern the significance or interpretation of the facts or predictions as to future facts.'[citation omitted] The D.C. Circuit made this distinction long ago in upholding the Federal Power Commission's approval of a settlement without a hearing: 'We believe that in each instance Penn Gas confuses contrary conclusions which might be drawn from accepted basic facts with contradictions in the basic facts themselves. We find no conflict in fundamental facts calling for a hearing . . . ."[citation omitted] Stated differently, "[a] dispute as to the inferences to draw from the facts, or as to the expert opinion most closely in conformity with the facts, is not sufficient."[citation omitted] Rather, to preclude certification of a settlement, there must be a dispute as

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11 *See PJM Interconnection, L.L.C.*, 182 FERC ¶ 61,194 at P 32 (2023).

12 *Id*.

> to the "basic underlying facts."[citation omitted] There is no such dispute here.

The Market Monitor provides ample evidence in this proceeding that the formula used to calculate the CRF values explicitly included current federal tax law. The formula was explained repeatedly to PJM stakeholders. The evidence that federal tax rates and depreciation provisions are explicitly included in the calculation of the current CRF values is undisputed. The settlement filed August 14, 2024 ("August 14th Settlement") that includes as settling parties the Indicated Suppliers and PJM concedes the fundamental issue that federal tax rates are explicitly included in the calculation of the CRF values and that the impact of the change in the federal tax rate created by the TCJA on the CRF values must be addressed in order to calculate just and reasonable rates. The problem with the August 14th Settlement is that it proposes to calculate the formula incorrectly, fails to address the impacts of the TCJA on the formula rates, and does so for reasons that are irrelevant, disputed and outside the scope of this proceeding.

2. Substantial Evidence Exists to Support the IMM Settlement.

The Commission has explained: "[T]o meet the substantial evidence requirement to support a finding, the courts have stated that the record must include "such relevant evidence as a reasonable mind might accept as adequate to support a conclusion."[13]

The record shows that the CRF values currently in the tariff were calculated based on a specific formula that included the federal tax code including tax rates and depreciation treatment for tax purposes but based on a tax code that was superseded effective January 1, 2018, as a result of the TCJA. There is no record supporting any change to the CRF formula

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[13] *Koch Gateway Pipeline Co.*, 75 FERC ¶ 61,132, 61457–61458 (1996) ("the substantial evidence requirement … has to do with the adequacy of the evidence relied upon to support a finding, whatever the kind of hearing. In order to meet the substantial evidence requirement to support a finding, the courts have stated that the record must include 'such relevant evidence as a reasonable mind might accept as adequate to support a conclusion.'"), citing Pierce v. Underwood, 487 U.S. 552, 564–65 (1988) ("Judicial review of agency action, the field at issue here, regularly proceeds under the rubric of 'substantial evidence' set forth in the Administrative Procedure Act, 5 U. S. C. § 706(2)(E). That phrase does not mean a large or considerable amount of evidence, but rather 'such relevant evidence as a reasonable mind might accept as adequate to support a conclusion.'").

in this proceeding. The CRF formula is correct but the tax code provisions included in the formula have not been correct since January 1, 2018. In this same proceeding, the Commission approved as just and reasonable the detailed inclusion of the CRF formula in the tariff, replacing the CRF table of values that are the product of that formula. The only question raised in the show cause order and in the hearing order is how to ensure that the CRF values are just and reasonable after the enactment of TCJA.

The record shows that the federal tax rates changed with the passage of TCJA from 36 percent to 21 percent and that bonus depreciation was added. The change to the tax code does not require any exercise of judgment to determine; the change concerns clearly defined objective values. The tax code results in clearly defined values that affect the calculation based on the change in the rate and the effect of the change in the depreciation provisions.

The IMM Offer is supported by substantial evidence. Substantial evidence supporting the proposed approach for relief requires such relevant evidence as a reasonable mind might accept as adequate to support a conclusion that the proposed approach accounts for the impact of the enactment of TCJA.

In the Affidavit of Dr. Joseph Bowring in support of the IMM Offer, the Market Monitor explains the basis for the IMM Offer.[14] The original source of the issue in this case was the failure of PJM to change the CRF values in the black start formula rate, effective January 1, 2018, to correctly reflect the changes in the tax code that resulted from the TCJA. PJM knew the inputs to the CRF and had a tariff defined responsibility to review the black start formula rate and its cost components for accuracy.[15] In PJM's review of and related report on the black start formula rates dated October 2019, PJM failed to recognize that the tax rate had changed effective January 1, 2018, and incorrectly stated the federal tax rate.[16]

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14 See IMM Offer, Bowring Affidavit.

15 OATT Schedule 6A para. 18 ("Every five years, PJM shall review the formula and its costs components set forth in this section 18, and report on the results of that review to stakeholders.").

16 *See* IMM Offer, Attachment G.

This case is about how to address PJM's failure to recognize the issue, and once the issue had been brought to their attention, PJM's failure to address the issue consistent with the tariff and in a timely manner. PJM could have filed to correct the tax rate included in the black start formula rate effective on January 1, 2018, at any time. Yet PJM failed to make any such filing. PJM proposed to change the formula rate only for black start generators that are scheduled for service after June 6, 2021. The result has been to substantially over charge PJM customers and to substantially over pay black start generators.

PJM, in this case, fails to take responsibility for that failure. PJM continues to delay a resolution of the issue that is based on the tax code and the tariff while attempting to blame the Market Monitor for delays.

C. The IMM Offer Should Be Approved By the Commission.

The Market Monitor recognizes that the IMM Offer is contested. A contested offer of settlement may only be approved based on its merits.[17] A contested settlement may be approved on its merits under one of the four approaches set forth in *Trailblazer Pipeline Company*.[18] Two of the approaches under *Trailblazer Pipeline Company* can be relied on for approval of the IMM Offer. The IMM Offer provides a just and reasonable resolution to the issue identified in the order setting this matter for hearing.[19]

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17 18 CFR § 385.602(h)(1) ("If the Commission determines that any offer of settlement is contested in whole or in part, by any party, the Commission may decide the merits of the contested settlement issues, if the record contains substantial evidence upon which to base a reasoned decision or the Commission determines there is no genuine issue of material fact.").

18 The four approaches for approving a settlement under *Trailblazer Pipeline Company* include: (i) addressing the contentions of the contesting party on the merits when there is any adequate record; (ii) approving a contested settlement as a package on the ground that the overall result of the settlement is just and reasonable; (iii) determining that the contesting party's interest is sufficiently attenuated such that the settlement can be analyzed under the fair and reasonable standard applicable to uncontested settlements when the settlement benefits the directly affected settling parties; or (iv) preserving the settlement for the consenting parties while allowing contesting parties to obtain a litigated result on the merits. *See Trailblazer Pipeline Company*, 85 FERC ¶ 61,345 (1998).

19 *See PJM Interconnection, L.L.C.*, 182 FERC ¶ 61,194 at P 32 ("[W]hether, as a result of changes from the TCJA, the existing CRF values result in a Capital Cost Recovery Rate for generating units that were selected to provide Black Start Service prior to June 6, 2021 that is unjust and unreasonable. While the record does not contain conclusive evidence that the existing CRF values include a 35% tax rate,

The record developed in this proceedings show that the IMM Offer's CRF values are just and reasonable under the first prong of *Trailblazer*. It follows that the result of the IMM Offer's CRF values are also just and reasonable if considered as a package, under *Trailblazer's* second prong, even if the Commission disagreed with some particular aspect of the approach to relief provided in the IMM Offer.

1. The Record Shows that the Current CRF Table Values Are Unjust and Unreasonable.

a. The Record Shows that the CRF Values Are Incorrect as a Result of the Tax Cuts in the TCJA.

Indicated Suppliers argue (at 6–7) that the Market Monitor has not shown that the current rate is unjust and unreasonable. But Indicated Suppliers claim that their August 14th Settlement resolves this proceeding on the merits. Indicated Suppliers ignore (at 8–9) the substantial and unrefuted evidence that the CRF values in the current formula include federal tax rates and, after January 1, 2018, use incorrect tax rates. The August 14th Settlement does not accurately account for the impacts of the TCJA, but it significantly changes the current rate. The current rates and the rates proposed in the August 14th Settlement cannot both be just and reasonable. Support for the August 14th Settlement is an implicit acknowledgement that the current rates do not account for the impacts of TCJA and must be changed.

b. Use of Objectively Incorrect CRF Values as Input to the Formula Rates Is Unjust and Unreasonable.

Indicated Suppliers argue (at 9–14) that even if "the IMM is correct regarding the calculation of the Legacy CRFs, this does not mean that those Legacy CRFs are now unjust and unreasonable as a result of the TCJA tax changes." The only true issue in this proceeding is whether the existing CRF values include an incorrect tax rate.[20] If the input is incorrect, and

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the Market Monitor has introduced sufficient evidence that those values may include a 35% tax rate, raising a disputed issue of material fact as to whether changes to the tax rate render the existing CRF values unjust and unreasonable. The import of the tax rate in the determination of the CRF value is a material fact that cannot be determined based on the existing record, which warrants setting the justness and reasonableness of the existing CRF values for hearing and settlement judge procedures.").

20 *See* 182 FERC ¶ 61,194 at P 32.

the correct input is known, it is unjust and unreasonable to include the incorrect input. The solution to the use of an incorrect input is axiomatic. The incorrect input must be corrected or properly accounted for. That use of the incorrect input creates a significant $89.7 million windfall to suppliers.[21]

2. The Relief Proposed in the IMM Offer Appropriately Accounts for the Impacts of the TCJA.

a. The Market Monitor's Approach Accounts for the Impacts of the TCJA.

Indicated Suppliers claim (at 15–18) that they are confused about how the Market Monitor's proposed relief operates. There is no reason for the confusion. The Market Monitor's approach was explained completely, sufficiently and correctly in the IMM Offer, and is further explained in this pleading.

b. Because the Market Monitor's Approach Avoids Retroactive Relief It Is Limited by the Refund Period and by the Completion of the Investment Recovery Period by Certain Suppliers.

Indicated Suppliers argue (at 22–23) that the relief proposed in the IMM Offer "ignores the statutory refund period." That is not correct. The IMM approach is limited by the refund period. Under the approach in the IMM Offer, no refunds apply by operation of law outside of the refund period even though the CRF values changed. This is an unavoidable limitation on the approach for relief in the IMM Offer.

c. No Adjustment Is Required to the TCJA 21 Percent Tax Rate.

Indicated Suppliers argue (at 24–25) that the TCJA 21 Percent Tax Rate should be adjusted. The 21 percent tax rate set by the TCJA is an objective fact. Indicated Suppliers have neither explained the need for nor supported any adjustment to that rate.

d. Once Approved, and the Relevant Time Periods Are Fixed, the Appropriate CRF Value for Each Supplier and the Resulting Formula Rate Can Be Readily Calculated.

Indicated Suppliers argue (at 24–25) that "the calculations in [the Market Monitor's confidential] Attachment T [showing CRF values and formula rate calculation using such values for each unit] cannot be considered determinative and substantial additional

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21 *See* Bowring Affidavit at 20:20–21.

proceedings would be required to implement the IMM's proposal." There is no reason why additional proceedings would be required as soon as all of the relevant parameters to perform the calculation are known. The application of the formula is objective. There is no reason why PJM could not perform the calculation. The unit specific data used in Attachment T are the capital investment amounts, service start date and the duration of the capital recovery term. These data points are required under the current rules and are known to PJM and the Market Monitor, and the black start providers have this information for their units.

3. The Rule Against Retroactive Ratemaking Does Not Bar the Relief Proposed in the IMM Offer.

PJM (at 17–19), Indicated Suppliers (at 18–22) and Staff argue (at 14–17) that the approach for relief proposed in the IMM Offer violates the filed rate doctrine. PJM argues (at 19), "These individualized CRFs would offset past over-recovery from units with time remaining on their Legacy CRF terms." PJM mischaracterizes how the relief proposed in the IMM Offer operates. The correct point of reference is the total return on investment over the full term of the CRF. No party cites to any requirement that a formula rate must be applied on an annual levelized basis. Application of the formula to calculate the correct CRF values for each resource is designed to help ensure that the correct recovery occurs over the duration of the CRF. There is no retroactive adjustment and violation under the IMM Offer. There is no complicated unit specific calculation. The only element that results in an "individualized" CRF is correctly selecting the start date and applying the formula.

The IMM Offer proposes an approach that avoids retroactive adjustments of any kind. The IMM Offer adjusts the applicable CRF values going forward from the start of the Commission's refund period, based on the capital recovered, which is a function of the service period start and end dates for each affected black start unit and the collected CRF value.[22] The end date, the point of reference for evaluating the recovery of investment, is in the future.

22 *See* Bowring Affidavit at 17:3–8.

The Market Monitor's settlement proposal does not include changing capital cost recovery rates for any prior period, other than for the Commission defined refund period.[23] The point of the refund period is to permit such changes. The CRF is a component of a capital cost recovery formula rate that defines total payments over a defined term.[24] If the CRF is overstated in the early years, regardless of the reason, it can be reduced in the later years in order to produce the intended result over the entire term of the CRF. That is not retroactive ratemaking as it does not require the repayment of payments made under a stated or filed rate.[25] The proposed going forward adjustment to the formula produces an outcome that is the only outcome consistent with the purpose of this specific formula rate, to provide recovery of all capital costs plus the defined return to both debt and equity investors over the defined term of the CRF.[26]

There is no bar to the use of a formula rate that sets the CRF values based on accelerated recovery of an investment. Indicated Suppliers cite to SFPP, L.P. v. FERC (*SFPP*), citing the case as an example showing that rates cannot indirectly operate retroactively.[27] In that case, the Commission did not allow the retention of a provision for adjusted deferred income taxes (ADIT) in a case where "where a pipeline's income tax allowance has been completely eliminated" for the purpose of "amortization of the sum that was once ADIT back to shippers in prospective rates."[28] In other words, the ADIT provision could not exist simply as a refund mechanism.

The treatment of ADIT described in *SFPP* is not analogous to what the IMM Offer proposes. The IMM Offer is instead consistent with what *SFPP* describes as allowed.

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23 *See id.* at 21:8–21.

24 *Id*.

25 *Id*.

26 *Id.*; *see also id.* 5:3–7:2 (describing special nature of the formula rate at issue in this case).

27 *See* Indicated Suppliers at 20 n.75, citing SFPP, L.P. v. FERC, 967 F.3d 788 (D.C. Cir. 2020) (*SFPP*).

28 *Id*. at 26–30.

SFPP explicitly distinguishes the use of ADIT provision when it is not used as a refund mechanism:

> Where an income tax allowance remains in the cost of service and there is excess ADIT resulting from a reduction in tax rates, it is appropriate to credit the cost of service to reflect that the pipeline currently needs to collect a lower level of tax expenses in rates to cover the tax liability for that year. Rather than returning the excess amounts to shippers related to past service, the pipeline's cost of service is adjusted on a going forward basis to reflect the fact that it now needs to collect less than what it anticipated to cover its future tax liabilities.[29]

What the IMM Offer proposes is analogous to what *SFPP* describes as acceptable and not retroactive ratemaking. *SFPP* shows that the relief proposed in the IMM Offer is not prohibited retroactive ratemaking.

Staff argues, citing a 2021 ISO New England case (*ISONE*) that the "type of relief" in the IMM Offer "is contrary to the rule against retroactive ratemaking, which 'prohibits the Commission from adjusting current rates to make up for a utility's over-or under-collection in prior periods.'"[30] *ISONE* relies on cases decided by the United States Court of Appeals for the D.C. Circuit, including Old Dominion Electric Cooperative v. FERC in 2018 (*ODEC*).[31]

ODEC illustrates Staff's and Indicated Suppliers' failure to distinguish between retroactive ratemaking and the application of formula rates. *ODEC* explains the limitations of the filed rate doctrine:

> In a similar vein, the rule against retroactive ratemaking 'prohibits the Commission from adjusting current rates to make up for a utility's over- or under-collection in prior periods.' Towns of Concord, 955 F.2d at 71 n.2. That otherwise categorical prohibition against retroactively charging rates that differ from those that were on file during the relevant time period yields in only two limited circumstances: (i) when a court invalidates the set rate as unlawful, and (ii) when the filed rate takes the form not of a number but of a

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29 *SFPP* at 23.

30 Staff at 13, citing *ISO New England Inc.*, 176 FERC ¶ 61,176 at P 12 (2021) (*ISONE*).

31 *ISONE* at 12, citing Old Dominion Elec. Coop. Inc. v. FERC, 892 F.3d 1223, 1227 (D.C. Cir. 2018) (*ODEC*); Towns of Concord, et al. v. FERC, 955 F.2d 67, 71 n.2 (D.C. Cir. 1992).

> formula that varies as the incorporated factors change over time. *See* West Deptford Energy, LLC v. FERC, 766 F.3d 10, 22-23, 412 U.S. App. D.C. 295 (D.C. Cir. 2014) (compiling cases). Neither of those exceptions apply to this case.[32]

In *West Deptford Energy*, the case cited in *ODEC*, the Court further explained:

> The Commission also invoked the so-called notice exception to the filed rate doctrine. We have said that the "filed rate doctrine simply does not extend to cases in which buyers are on adequate notice that resolution of some specific issue may cause a later adjustment to the rate being collected at the time of service."[citation omitted]. Unfortunately, "[o]ur decisions on the necessary notice have not been altogether clear."[citation omitted]. For the most part, however, the notice exception has been confined to two scenarios.
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> First, it permits the filing of tariffs that provide a formula for calculating rates, rather than a specific rate number. This court had held that such a "formula itself is the filed rate that provides sufficient notice to ratepayers" to comport with the Federal Power Act's open-filing requirements, and that the objectivity of formulae ensures evenhandedness, predictability and stability in rates.[Citation omitted]…[33]

The "notice" exception concerning formula rates identified in *ODEC* and *West Deptford* apply in this case. Paragraph 18 states a formula rate, not a specific dollar value. The CRF values are a component of the formula and are not dollar values. The IMM Offer proposes prospective relief. If, however it is determined that the relief proposed in the IMM Offer means correctly calculating the Capital Recovery Rate now and applying that later adjustment to the rate collected at the time of service, then the prohibition against retroactive ratemaking rate does not apply.

4. Additional Issues.

a. There Is No Adverse Impact on Incentives for Investment in Black Start Capability.

Indicated Suppliers argue (at 24), that accepting the relief proposed in the IMM Offer would discourage investment in black start capability. PJM Witness Bryson (at 7) states

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32 *ODEC* at 1227.

33 West Deptford Energy, LLC v. FERC, 766 F.3d 10, 22–23 (D.C. Cir. 2014).

concerns that "such minimal compensation" under the IMM Offer would affect PJM's ability to attract participation in future request for proposals (RFPs) for black start service. The statements are unfounded and demonstrate a lack of understanding of the IMM Offer and how the black start capital recovery rate works. The CRF is calculated so that the recovery rate will cover the income tax liabilities, provide for the return of the investment and provide the defined rate of return on the investment. At no point in this process has the Market Monitor suggested that these defining conditions of the CRF not be fulfilled, and the IMM Offer adheres to the defining conditions. The profitability of investments in black start service under the IMM Offer is greater than what it would have been if the TCJA had not been implemented and greater than what is defined in the OATT. Such compensation cannot accurately be characterized as "minimal compensation." Only the federal income tax rate has changed for the CRF applicable to generators that began service prior to 2018. For generators that began service after January 1, 2018, and before June 6, 2021, the CRFs in the IMM Offer reflect the effects of bonus depreciation eligibility in addition to the updated federal income tax rate. The IMM Offer does not alter the financial parameters and therefore does not alter the return to black start investments. These are exactly the same financial parameters that PJM stated just last week did not need to be updated due to the success of recent black start service RFPs.[34]

b. *Suppliers Entering Service After TCJA Became Effective Are Treated Fairly Under the IMM Offer.*

Indicated Suppliers argue (at 24) that the relief proposed in the IMM Offer is unfair to suppliers that "submitted offers and were accepted by PJM after the effective date of the TCJA tax changes." The Market Monitor has shown that the black start service providers that began service after January 1, 2018, and prior to June 6, 2021, had the opportunity to achieve profits well in excess of the intended 12 percent return on equity. If the black start service providers had matched the financial structure assumed in the CRF calculations (50/50 debt to equity

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34 *Review of Black Start Formula and Cost Components* (*see* Section V.A. at 13), PJM Interconnection, L.L.C. (October 11, 2024) <*https://pjm.com/-/media/markets-ops/ancillary/black-start-service/review-of-black-start-formula-and-cost-components.ashx*>.

with debt at 7 percent interest) and had taken advantage of bonus depreciation, the rate of return to equity investors would have exceeded 60 percent on the five year capital recovery term.

c. The Relief Proposed in the IMM Offer Should Not Confuse Any Qualified Reviewer.

There appears to be inexplicable confusion regarding the IMM Offer. PJM Witness Boyle says (at 7) the "precise method" the IMM used to calculate the CRFs is unclear. The Indicated Suppliers state (at 4) that the IMM Offer is the same recommendation the IMM made three years ago.

The IMM Offer retains, from the recommendation made in November 2021, the position that the updated CRF should account for the capital cost recovery that has already taken place. The IMM Offer establishes September 1, 2021, as the effective date for calculating the updated CRF values and January 1, 2025, as the effective date for the revenue payments to reflect the updated CRF values. The IMM Offer also includes provisions for calculating refunds during the FERC defined 15 month refund period. To further assist in understanding the IMM Offer, the Market Monitor has updated the CRF Technical Reference, attached as Attachment U.

The CRF updates included in the IMM Offer are calculated using the CRF formula (2.1) on page 21 of Attachment U.[35] The parameter inputs to the formula are described in Table 2-1 on page 20. The only unit specific input to the CRF formula is the black start service start date. Section 2.1 of Attachment U includes several examples and Section 2.3 provides a derivation of the CRF formula in (2.1) of Attachment U.

Consider the following example. A black start unit began service on January 1, 2016, with a $10 million capital investment to be recovered over a ten year period. The applicable CRF when the unit began service was 0.198 and the annual revenue rate was $1,980,000. The CRF update in the IMM Offer is calculated assuming an effective date of September 1, 2021.

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35 *See* Comments of the Independent Market Monitor for PJM, Docket No. EL21-91-000 (November 11, 2021).

Using the same financial parameters and income tax assumptions used to produce the original CRF values, with an update to the federal income rate consistent with the TCJA, the CRF produced by the CRF formula (2.1) of Attachment U is 0.147170 and the updated annual revenue rate is $1,471,697.[36] Table I-1 shows the associated cash flow summary. In Table I-1, the effective income tax rate changes in year 3 of the capital recovery period and the annual revenue rate changes in year 6.[37] [38] The CRF defined by the IMM Offer takes into account the accelerated recovery that occurred prior to September 1, 2021, and adjusts the going forward revenue so that the capital recovery adheres to the original intent, namely that the capital recovery provides for the income tax liabilities and the return on and of the investment over the term of the CRF. Table I-1 shows that this is indeed the case.

In fact, the revenue payments did not change on September 1, 2021, to reflect the updated CRF. Under the IMM Offer, the CRF calculated with an effective date of September 1, 2021, will be reflected in the revenue payments to black start units beginning on January 1, 2025. Table I-2 shows the corresponding cash flow summary.

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36 *See* Table 1-3 (page 9) in Attachment U for a list of the parameter assumptions.

37 The effective income tax rate changes from 41.76 percent to 28.11 percent.

38 The revenue in year 6 reflects eight months at the original CRF and four months at the updated CRF.

Table I-1 Cash flow summary with updated revenue payments beginning September 1, 2021[39]

Capital Recovery Year	1	2	3	4	5	6	7	8	9	10
Revenue	$1,980,000	$1,980,000	$1,980,000	$1,980,000	$1,980,000	$1,810,566	$1,471,697	$1,471,697	$1,471,697	$1,471,697
Depreciation	$500,000	$950,000	$855,000	$770,000	$693,000	$623,000	$590,000	$590,000	$591,000	$590,000
Interest on debt	$172,040	$313,868	$287,665	$259,627	$229,626	$197,525	$163,177	$126,425	$87,100	$45,023
Income Tax	$546,204	$299,057	$235,375	$267,150	$297,228	$278,300	$201,976	$212,307	$223,080	$235,189
Return on equity (ROE)	$291,503	$545,510	$491,843	$424,094	$352,028	$274,923	$206,627	$161,641	$112,496	$58,746
Revenue in excess of taxes, interest and ROE	$970,253	$821,565	$965,118	$1,029,130	$1,101,119	$1,059,817	$899,916	$971,324	$1,049,020	$1,132,738
Repayment of debt principal	$516,167	$374,339	$400,543	$428,581	$458,582	$490,683	$525,030	$561,782	$601,107	$643,185
Repayment of equity investment	$454,086	$447,226	$564,575	$600,549	$642,537	$569,134	$374,886	$409,541	$447,913	$489,554
Debt Remaining	$4,483,833	$4,109,493	$3,708,950	$3,280,369	$2,821,787	$2,331,105	$1,806,074	$1,244,292	$643,185	$0
Equity Remaining	$4,545,914	$4,098,688	$3,534,113	$2,933,565	$2,291,028	$1,721,894	$1,347,008	$937,467	$489,554	$0
Excess Revenue to equity investors	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
After tax cash flow to equity investors	$745,588	$992,736	$1,056,417	$1,024,642	$994,564	$844,058	$581,513	$571,182	$560,409	$548,300

Table I-2 Cash flow summary with updated revenue payments beginning January 1, 2025

Capital Recovery Year	1	2	3	4	5	6	7	8	9	10
Revenue	$1,980,000	$1,980,000	$1,980,000	$1,980,000	$1,980,000	$1,980,000	$1,980,000	$1,980,000	$1,980,000	$1,471,697
Depreciation	$500,000	$950,000	$855,000	$770,000	$693,000	$623,000	$590,000	$590,000	$591,000	$590,000
Interest on debt	$172,040	$313,868	$287,665	$259,627	$229,626	$197,525	$163,177	$126,425	$87,100	$45,023
Income Tax	$546,204	$299,057	$235,375	$267,150	$297,228	$325,928	$344,860	$355,191	$365,964	$235,189
Return on equity (ROE)	$291,503	$545,510	$491,843	$424,094	$352,028	$274,923	$192,010	$101,420	$1,198	$0
Revenue in excess of taxes, interest and ROE	$970,253	$821,565	$965,118	$1,029,130	$1,101,119	$1,181,623	$1,279,952	$1,396,964	$1,525,738	$1,191,485
Repayment of debt principal	$516,167	$374,339	$400,543	$428,581	$458,582	$490,683	$525,030	$561,782	$601,107	$643,185
Repayment of equity investment	$454,086	$447,226	$564,575	$600,549	$642,537	$690,941	$754,922	$835,182	$9,984	$0
Debt Remaining	$4,483,833	$4,109,493	$3,708,950	$3,280,369	$2,821,787	$2,331,105	$1,806,074	$1,244,292	$643,185	$0
Equity Remaining	$4,545,914	$4,098,688	$3,534,113	$2,933,565	$2,291,028	$1,600,087	$845,165	$9,984	$0	$0
Excess Revenue to equity investors	$0	$0	$0	$0	$0	$0	$0	$0	$914,647	$548,300
After tax cash flow to equity investors	$745,588	$992,736	$1,056,417	$1,024,642	$994,564	$965,864	$946,933	$936,601	$925,828	$548,300

The annual revenue remains at the original rate through year 9 of the capital recovery. In year 10, the revenue rate is updated to reflect the updated CRF, 0.147170. Table I-2 shows that equity investment has been repaid by the end of year 9 of the recovery period and excess recovery exceeds $1.5 million in the final two years of the capital recovery, even with the revenue reduction in year 10. The refund, required in the IMM Offer, is equal to the difference between the revenue at the original CRF and the updated CRF over the 15 month period

[39] Section 1.2 (at page 12) in the Attachment U explains how the entries in cash flow summary are determined.

beginning on September 1, 2021. In this example, the refund would equal $635,379.[40] The refund recovers a portion of the excess revenue but clearly not all of the excess recovery and the equity investors will still earn a return in excess of the intended 12 percent rate. Interest on the refund is calculated in accordance with Title 18 in the Code of Federal Regulations.[41] [42] The interest on refund for this example, using monthly prime rates through August 2024, is $138,980.

A second example shows the impact of bonus depreciation on the CRF rate in the IMM Offer. Consider a black start unit that began service on January 1, 2021, with a $10 million capital investment to be recovered over a five year period. Such an investment would have been eligible for bonus depreciation and the CRF update calculation in the IMM Offer reflects this eligibility. This example differs from the previous example in that the original revenue payment for this resource is incorrect because it is based on the tax code in place prior to TCJA. This resource would have been assigned a CRF of 0.363 under the PJM black start capital cost recovery rules in place prior to TCJA and that are still in place. The CRF value of 0.363 was calculated assuming a 36 percent federal income tax rate and 15 MACRS depreciation. Formula (2.1) in Attachment U, assuming an effective date of September 1, 2021, results in a CRF update of 0.218295. Table I-3 shows the cash flow summary under the assumption that the revenue payment was updated on September 1, 2021. Under this assumption there is no excess recovery and there is no under recovery.

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40 The refund calculation is ($1,980,000 - $1,471,697) x (15/12).

41 Code of Federal Regulations, Title 18, Chapter I, Subchapter B, Part 35, Subpart C §35.19a.

42 The interest calculations will have to be updated to reflect the interest rates in the last four months of 2024.

Table I-3 Cash flow summary with updated revenue payments beginning September 1, 2021

Capital Recovery Year	1	2	3	4	5
Revenue	$3,147,649	$2,182,946	$2,182,946	$2,182,946	$2,182,946
Depreciation	$10,000,000	$0	$0	$0	$0
Interest on debt	$172,040	$279,521	$216,565	$149,202	$77,124
Income Tax	($1,974,556)	$535,053	$552,750	$571,686	$591,947
Return on equity (ROE)	$291,503	$161,783	$124,916	$85,749	$44,155
Revenue in excess of taxes, interest and ROE	$4,658,662	$1,206,590	$1,288,716	$1,376,310	$1,469,721
Repayment of debt principal	$1,006,850	$899,370	$962,326	$1,029,688	$1,101,767
Repayment of equity investment	$3,651,812	$307,221	$326,390	$346,622	$367,955
Debt Remaining	$3,993,150	$3,093,780	$2,131,455	$1,101,767	$0
Equity Remaining	$1,348,188	$1,040,967	$714,576	$367,955	$0
Excess Revenue to equity investors	$0	$0	$0	$0	$0
After tax cash flow to equity investors	$3,943,315	$469,003	$451,306	$432,371	$412,110

Table I-4 shows the cash flows assuming the revenue payments reflect the updated CRF value beginning on January 1, 2025, as proposed in the IMM Offer. Only the final year of recovery, year 5, reflects the updated revenue payment. The equity investors would have been fully compensated for the return of and on their investment by the end of year 2 and years 2 through 5 shows over $3.7 million in excess revenue to the equity investors.

Table I-4 Cash flow summary with updated revenue payments beginning January 1, 2025

Capital Recovery Year	1	2	3	4	5
Revenue	$3,630,000	$3,630,000	$3,630,000	$3,630,000	$2,182,946
Depreciation	$10,000,000	$0	$0	$0	$0
Interest on debt	$172,040	$279,521	$216,565	$149,202	$77,124
Income Tax	($1,838,968)	$941,820	$959,517	$978,452	$591,947
Return on equity (ROE)	$291,503	$120,171	$0	$0	$0
Revenue in excess of taxes, interest and ROE	$5,005,425	$2,288,489	$2,453,919	$2,502,346	$1,513,876
Repayment of debt principal	$1,006,850	$899,370	$962,326	$1,029,688	$1,101,767
Repayment of equity investment	$3,998,575	$1,001,425	$0	$0	$0
Debt Remaining	$3,993,150	$3,093,780	$2,131,455	$1,101,767	$0
Equity Remaining	$1,001,425	$0	$0	$0	$0
Excess Revenue to equity investors	$0	$387,694	$1,491,593	$1,472,657	$412,110
After tax cash flow to equity investors	$4,290,077	$1,509,290	$1,491,593	$1,472,657	$412,110

The refund under the IMM Offer refund provision would be $1,808,817 and the interest on the refund would be $395,654.[43] As in the first example, the refund recovers a

43 The refund calculation is ($3,630,000 - $2,182,946) x (15/12).

portion of the excess revenue but clearly not all of the excess recovery and the equity investors will still earn a return in excess of the intended 12 percent rate. This illustrates that the IMM Offer is a settlement proposal and does not recapture all of the excess returns.

Except for the units with a 20 year recovery term, these examples, while hypothetical, illustrate the situations of actual pre June 6 black start resources that are currently receiving capital recovery payments defined by the outdated and incorrect CRF rates. These examples show that the IMM Offer CRFs strike a balance between allowing over recovery to continue and setting the CRF so that no over recovery occurs. The IMM Offer, and all recommended remedies by the Market Monitor throughout this docket, sets the recovery rate at a value that provides for the income tax liabilities and the return of and required return on the capital investment. The IMM Offer is a settlement proposal. The IMM Offer unavoidably does not require that customers be made whole for all the excess returns paid based on continued use by PJM of the incorrect CRF rates.

d. The Flow to Equity Model.

There is a similarly inexplicable confusion regarding the flow to equity (FTE) approach and the weighted-average cost of capital (WACC) approach. Indicated Suppliers state (at 9) that "Dr. Bowring has applied this FTE label inconsistently to a range of models with different assumptions." Indicated Suppliers Witness Kimbrough (at 7) says the Market Monitor's explanation for the calculation of the original CRF values is "incomplete and contradictory." In PJM Exhibit No. PJM-0008, PJM incorrectly referred to the PJM settlement CRF values as "FTE/Levelized" when in fact there were calculated using a formula derived from the WACC approach. The statements by Indicated Suppliers are incorrect and arise from a lack of understanding of the FTE model.

Both the FTE model and the WACC approach must assume a structure for the repayment to the debt and equity investors, and each approach must include a timing assumption regarding the receipt of the various components that make up the after tax cash flow. Witness Kimbrough's Table 1 through Table 4 (at 6) merely compare the interest calculations between an end of year debt payment approach and the half year convention where the debt payments are assumed to occur at the midpoint of the year.

The WACC approach formula, included in Schedule 6A to the OATT and applicable to black start units that were scheduled for service after June 6, 2021, assumes a constant debt to equity ratio throughout the capital recovery and assumes the half year convention for the timing of all components of the cash flow. The cash flow components consist of revenue, income tax payments, payments to the debt provider, and repayment on and return of the equity investment. A description of this approach and a derivation of the formula is provided in Section 1.2 of Attachment V.

The FTE approach, used to calculate the original CRF values that are currently being used for black start units that were scheduled for service prior to June 6, 2021, assumes a mortgage style debt payment structure. The debt to equity ratio is not constant under this approach. Regarding the timing of the cash flow components, the debt payments assume an end of year payment structure and the other components (revenue, income tax and repayment and return to equity investors) assume the half year convention. A description of this FTE model with this hybrid approach to the cash flow components is included in Section 1.3 of Attachment U. Section 1.3 of Attachment U also includes a formula corresponding to this approach and a comparison of the formula to the simulations that produced the original CRF values.

The calculation of the updated CRF values in the IMM Offer use the FTE approach with a mortgage style debt payment structure and the half year convention for all cash flow components. The Market Monitor recommends that all PJM CRF calculations use this approach. The Market Monitor concluded that is the better approach because the FTE model correctly captures the impacts of excess payments. The FTE model reflects that the excess revenue flows to the equity investors and not to both debt holders and equity investors. The WACC model distributes the excess revenue both to the debt holders and the equity investors so as to maintain a constant debt to equity ratio.

The Market Monitor recommends using the half year convention for all components. A description of the Market Monitor's calculation method, the FTE approach with mortgage style debt payments and the half year convention for all cash flow components, is provided in Section 1.2 of Attachment U. A description of the CRF update calculation is described in Section 2.1 and derived in Section 2.3 of Attachment U. An important mathematical result, that validates the consistency of the CRF update process, is that the CRF update formula, (2.1) in Attachment U, is an extension of the CRF formula (1.7) in Section 1.2 of Attachment U. Formula (1.7) is applicable to new investments and is the FTE analogue of the WACC formula in the PJM OATT. Section 2.2 in Attachment U shows that the CRF update formula (2.1) reduces to formula (1.7) under the appropriate parameter settings for an investment that is just beginning the capital recovery term.

Witness Kimbrough's states (at 12) that the Market Monitor "offered conflicting accounts" of which model was used to calculate the original CRF values. Witness Kimbrough cites the Market Monitor's response to VIS-IMM-2.3(b). The Market Monitor's response to VIS-IMM-2.3(b) is correct. The CRF formula that VIS-IMM-2.3(b) references is for the FTE model with the half year convention for all cash flow components and it was not used to calculate the original CRF values. The original CRF values were calculated using the FTE model with a hybrid approach to the timing of the cash flow components. The debt payments were assumed to have occurred at the end of year and all other components assumed the half year convention. Section 1.3 in Attachment U provides additional details on the calculation of the original CRF values using the hybrid approach.

e. The IMM CRF Formula.

Staff (at 13) characterizes the IMM CRF as "unit-specific rates calculated by a third party." PJM Witness Boyle (at 4, item 9) says the IMM's "unit-specific CRFs" are not consistent with the stated CRFs in the PJM tariff. These assertions are incorrect in addition to being incomplete and misleading.

The CRF values used in the IMM Offer were calculated using the CRF update formula which is described in Section 2.1 and derived in Section 2.3 of Attachment U. The CRF values are not uniquely calculated by the Market Monitor. The Market Monitor, to which Staff refers to as the "third party," did put the calculation in a spreadsheet and populated the parameters. The proposal is a formula that can be applied by PJM or anyone else. The rate is calculated following the formula and could be done by anyone with access to the data. The CRF rate does not depend on any unit characteristics. The CRF update is unit specific only because the rate depends on the service start date. Two units with the same start date and the same capital recovery term, will have exactly the same updated CRF value. The service start date is necessary to determine the capital recovery that has occurred as of the date the CRF is updated.

II. CONCLUSION

Because the IMM Offer provides a just and reasonable resolution to the issue raised in this proceeding and is supported by substantial evidence, it should be certified and approved.

Respectfully submitted,

Jeffrey W. Mayes

Joseph E. Bowring
Independent Market Monitor for PJM
President
Monitoring Analytics, LLC
2621 Van Buren Avenue, Suite 160
Eagleville, Pennsylvania 19403
(610) 271-8051
joseph.bowring@monitoringanalytics.com

Jeffrey W. Mayes

General Counsel
Monitoring Analytics, LLC
2621 Van Buren Avenue, Suite 160
Eagleville, Pennsylvania 19403
(610) 271-8053
jeffrey.mayes@monitoringanalytics.com

John Hyatt
Senior Economist
Monitoring Analytics, LLC
2621 Van Buren Avenue, Suite 160
Eagleville, Pennsylvania 19403
(610) 271-8050
john.hyatt@monitoringanalytics.com

Dated: October 21, 2024

CERTIFICATE OF SERVICE

I hereby certify that I have this day served the foregoing document upon each person designated on the official service list compiled by the Secretary in this proceeding.

Dated at Eagleville, Pennsylvania,
this 21st day of October, 2024.

Jeffrey W. Mayes
General Counsel
Monitoring Analytics, LLC
2621 Van Buren Avenue, Suite 160
Eagleville, Pennsylvania 19403
(610) 271-8053
jeffrey.mayes@monitoringanalytics.com

Attachment U

Capital Recovery Factors (CRF)
Flow to Equity Approach
Technical Reference

Monitoring Analytics, LLC

Revised October 21, 2024

Table of Contents

1 The Basics of CRF

A capital recovery factor (CRF) is used to convert the principal of a capital investment (K) into an equivalent stream of uniform payments. A typical CRF formula found in engineering economics textbooks is given in equation (1.1).[1]

(1.1)

$$CRF = \frac{r(1+r)^N}{(1+r)^N - 1}$$

Variable r is the rate of return and N is the term (in years) over which the investment will be recovered. At the end of each year during the recovery term, the investor receives a payment equal to the product of the CRF and the investment. For example, consider a $1 million dollar investment. Assume the rate of return is 12 percent and the capital recovery term is five years. The CRF value given by (1.1) is 0.277410 and the uniform annual payment, to be paid at the end of each year of the five year recovery period, is $277,410. Table 1-1 shows the corresponding cash flow. The first payment is received at the end of the first capital recovery year. The payment covers the return on investment equal to $120,000, leaving $157,410 for repayment of the investment principal. The return on investment at the end of the first year is the product of the investment amount ($1 million) and the rate of return (12 percent). The return on investment at the end of year 2 is the product of the rate of return and the outstanding principal at the end of year 1. At the end of year 2, the difference between the annual revenue and the return on investment is $176,299 which goes toward repayment of the investment principal. The cash flow components for years 3 through 5 are analogous to the year 2 cash flow. At the end of year 5, the investment principal has been returned. The cash flow summary in Table 1-1 shows that the annual revenue determined by the CRF in (1.1) has provided for the return on and return of the $1 million investment.

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[1] For example, see pages 21-22 in "Economic Evaluation and Investment Decision Methods," Stermole, F.J. and Stermole, J.M. (1993).

Table 1-1 Investment cash flow for the basic CRF

Capital Recovery Year	1	2	3	4	5
Revenue	$277,410	$277,410	$277,410	$277,410	$277,410
Return on investment	$120,000	$101,111	$79,955	$56,260	$29,722
Repayment of investment principal	$157,410	$176,299	$197,455	$221,149	$247,687
Outstanding investment principal	$842,590	$666,291	$468,837	$247,687	$0

To derive (1.1) the CRF is denoted by c and the annual payment is defined as cK, the product of the CRF and the investment capital. The value of c is determined by requiring the present value of the annual payments is equal to the investment capital. The following equation captures this requirement,

$$K = \sum_{j=1}^{N} \frac{cK}{(1+r)^j}$$

which can be restated as

$$K = cK \sum_{j=1}^{N} \left(\frac{1}{1+r}\right)^j .$$

The summation in the equation is a finite geometric series. A formula for the sum of a finite geometric series is given by (3.1) in the appendix. Using (3.1) with $H = 1$, $W = N$ and $v = 1/(1+r)$ yields

$$\sum_{j=1}^{N} \left(\frac{1}{1+r}\right)^j = \frac{(1+r)^N - 1}{r(1+r)^N} .$$

Upon replacing the summation in the present value equation

$$K = cK \left(\frac{(1+r)^N - 1}{r(1+r)^N}\right)$$

and solving for c produces (1.1).

1.1 After tax CRF

The CRF value in (1.1) is a before tax calculation since it does not account for income taxes. The revenue that results from a capital investment is taxable income. The payment obtained by multiplying the capital investment amount K by the CRF in equation (1.1) would be too low in cases where the revenue is taxable. The goal, in the presence of taxes, is to have a CRF for which the product of the CRF and the investment K yields an annual payment that will provide the necessary and sufficient level of revenue to cover the investors' annual income tax payments, and the return on and return of the capital investment. The CRF and the associated annual revenue payment, can be determined by solving an equation where the present value of the after tax cash flows resulting from the annual revenue payments is equal to the initial capital investment.

The composition of the after tax cash flow is dependent upon the capital budgeting model. The flow to equity (FTE) model was used to develop the original CRF for PJM Black Start Service.[2] The FTE approach discounts the after tax cash flow to the equity investor at the assumed rate of return on equity. The CRF must satisfy the following present value equation,

$$EK = \sum_{j=1}^{N} \frac{CF_j}{(1 + r_e)^j} \ .$$

Parameter E is the percent of capital provided by equity investors and EK is the equity portion of the capital investment K. CF_j is the after tax cash flow to the equity investor for year j, r_e is the rate of return on equity and the revenue, tax and debt payments are assumed to occur at the end of the year. The model parameters are defined in Table 1-2. In the FTE model, the after tax cash flow is equal to the revenue net of taxes and the debt payment, and the tax calculation includes an offset for depreciation and interest on the debt. The after tax cash flow for year j is

$$\begin{aligned} CF_j &= cK - \left(cK - \delta_j K - I_j\right)s - P \\ &= cK(1 - s) + \delta_j K s + I_j s - P \end{aligned}$$

where c is the CRF, K is the total capital investment including debt and equity, I_j is the interest portion of the debt payment P, δ_j is the depreciation factor and s is the effective tax rate. Upon replacing CF_j in the present value equation

2 Additional details on the flow to equity approach can be found in Section 17.2 in "Corporate Finance," Ross, Westerfield, Jaffe, 4th Edition, 1996.

$$E \cdot K = cK(1-s)\sum_{j=1}^{N}\frac{1}{(1+r_e)^j} + Ks\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^j} + s\sum_{j=1}^{N}\frac{I_j}{(1+r_e)^j} - P\sum_{j=1}^{N}\frac{1}{(1+r_e)^j}.$$

Using (3.1)

$$\sum_{j=1}^{N}\frac{1}{(1+r_e)^j} = \frac{(1+r_e)^N - 1}{r_e(1+r_e)^N}$$

and substituting into the previous equation results in

$$E \cdot K = cK(1-s)\left(\frac{(1+r_e)^N - 1}{r_e(1+r_e)^N}\right) + Ks\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^j} + s\sum_{j=1}^{N}\frac{I_j}{(1+r_e)^j} - P\left(\frac{(1+r_e)^N - 1}{r_e(1+r_e)^N}\right).$$

Solving for c yields

(1.2)

$$c = \frac{r_e(1+r_e)^N}{(1-s)[(1+r_e)^N - 1]}\left\{E - s\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^j} - \frac{s}{K}\sum_{j=1}^{N}\frac{I_j}{(1+r_e)^j} + \frac{P}{K}\frac{(1+r_e)^N - 1}{r_e(1+r_e)^N}\right\}.$$

Table 1-2 Parameter descriptions for the FTE capital budgeting model

Parameter	Description
K	Capital investment (including debt and equity)
E	Equity funding percent
r_e	Rate of return on equity
r_d	Debt interest rate
P	Debt payment
I_j	Interest portion of debt payment in year *j*
s	Effective income tax rate
N	Capital investment recovery term
δ_j	Depreciation factor for year *j*

Formulas for the debt payment and interest portion of the debt payment, for debt with a term of N years and assuming end of year debt payments, are provided in (3.2). Equation (3.4) provides a formula for the sum of discounted interest payments. Using (3.4) with $v = 1/(1+r_e)$,

(1.3)

$$\sum_{\mathrm{j=1}}^{\mathrm{N}}\frac{\mathrm{I_j}}{(1+\mathrm{r_e})^{\mathrm{j}}}=(1-E)K\frac{r_d}{(1+r_d)^N-1}\left[(1+r_d)^N\left(\frac{(1+r_e)^N-1}{r_e(1+r_e)^N}\right)-\frac{(1+r_e)^N-(1+r_d)^N}{(r_e-r_d)(1+r_e)^N}\right].$$

Replacing the sum of discounted interest payments in equation (1.2) with the expression in (1.3), and using (3.2) to replace P yields the CRF formula in (1.4).[3]

(1.4)

$$c=\frac{r_e(1+r_e)^N}{(1-s)[(1+r_e)^N-1]}\left\{E-s\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^j}\right.$$
$$-(1-E)s\frac{r_d}{(1+r_d)^N-1}\left[(1+r_d)^N\left(\frac{(1+r_e)^N-1}{r_e(1+r_e)^N}\right)-\left(\frac{(1+r_e)^N-(1+r_d)^N}{(r_e-r_d)(1+r_e)^N}\right)\right]$$
$$\left.+(1-E)\left(\frac{r_d(1+r_d)^N}{(1+r_d)^N-1}\right)\left(\frac{(1+r_e)^N-1}{r_e(1+r_e)^N}\right)\right\}.$$

Substituting the parameter values shown in Table 1-3 into the CRF formula, assuming a five year capital recovery period and straight line depreciation yields a CRF of 0.287314. With a capital investment of $1 million, the annual payment is $287,314.

Table 1-4 provides a cash flow summary for a $1 million capital investment with a five year cost recovery period that uses straight line depreciation. The revenue for each year, equal to the product of the CRF and the capital investment amount is $287,314. The tax payment for each year is equal to the effective income tax rate times the revenue net of depreciation and the interest portion of the debt payment. The interest payment in year 1 is equal to the product of the debt interest rate and the initial debt of $500,000, and the return on equity in year 1 is equal to the product of the rate of return on equity and the initial equity investment of $500,000.

[3] For debt principal (D) is equal to $(1-E)K$.

Table 1-3 Financial parameter and tax assumptions[4]

Parameter	Parameter Value
Equity funding percent	50.00%
Debt funding percent	50.00%
Rate of return on equity	12.00%
Interest rate on debt	7.00%
Federal income tax rate	36.00%
State income tax rate	9.00%
Effective income tax rate	41.76%

After accounting for the tax payment, the debt payment and return on equity in year 1, $83,523 is available for repayment of the equity investment. The equity investment remaining is $416,477 at the end of year 1. The year 2 interest on debt is the product of the debt interest rate and the outstanding debt at the end of year 1.[5] The year 2 return on equity is the product of the rate of return on equity and the equity investment remaining at the end of year 1. Repayment to equity investors is $91,004 in year 2. The cash flows for years 3 through 5 are analogous to the year 2 cash flow.

4 The effective tax rate (parameter s in the formula) is equal to *State Tax Rate + Federal Tax Rate x (1-State Tax Rate).*

5 The outstanding or remaining investment refers to the investment principal that has not been repaid.

Table 1-4 Cash flow summary for 5 year, $1 million investment with straight line depreciation[6]

Capital Recovery Year	1	2	3	4	5
Revenue	$287,314	$287,314	$287,314	$287,314	$287,314
Depreciation	$200,000	$200,000	$200,000	$200,000	$200,000
Interest on debt	$35,000	$28,914	$22,402	$15,434	$7,978
Income Tax	$21,847	$24,388	$27,108	$30,017	$33,131
Return on equity (ROE)	$60,000	$49,977	$39,057	$27,152	$14,168
Revenue in excess of taxes, interest and ROE	$170,468	$184,035	$198,748	$214,711	$232,037
Repayment of debt principal	$86,945	$93,032	$99,544	$106,512	$113,968
Repayment of equity investment	$83,523	$91,004	$99,205	$108,199	$118,070
Debt Remaining	$413,055	$320,023	$220,479	$113,968	$0
Equity Remaining	$416,477	$325,474	$226,269	$118,070	$0

After the final revenue payment in year 5, the remaining equity investment and the outstanding debt are reduced to $0. Summing horizontally across the debt repayment row and the equity repayment row produces $500,000 for each row, reflecting the 1:1 debt to equity ratio in Table 1-3. This example illustrates that the revenue payment determined by the CRF provides the necessary and sufficient annual revenue to pay the taxes associated with the revenue payment as well as the required return on and return of the capital investment.

1.2 Half Year Convention

The revenue and tax payments would likely be made on a monthly or quarterly basis rather than occurring at the end of the year. A better model with respect to the timing of the revenue and tax payments is obtained by assuming the revenue and tax payments occur at the midpoint of each year. To derive a CRF corresponding to midyear revenue and tax payments, the present value equation from the previous section is modified to reflect the new timing assumption. Each after tax cash flow amount is assumed to occur a half year earlier than in the previous model. The revised present value equation is

$$K = \sum_{j=1}^{N} \frac{CF_j}{(1+r_e)^{j-0.5}} ,$$

[6] FTE model with end of year revenue and tax payments.

or equivalently,

$$\mathrm{K} = \sqrt{1+r_e}\sum_{\mathrm{j}=1}^{\mathrm{N}} \frac{CF_j}{(1+r_e)^{\mathrm{j}}} \ .$$

Making the substitution,

$$CF_j = cK - \left(cK - \delta_j K - I_j\right)s - P$$

and solving for c yields equation (1.5).

(1.5)

$$c = \frac{r_e(1+r_e)^N}{(1-s)[(1+r_e)^N - 1]}\left\{\frac{E}{\sqrt{1+r_e}} - s\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^j} - \frac{s}{K}\sum_{j=1}^{N}\frac{I_j}{(1+r_e)^j} + \frac{P}{K}\frac{(1+r_e)^N - 1}{r_e(1+r_e)^N}\right\}.$$

Formulas for the debt payment and interest portion of the debt payment, assuming a term of N years and the half year convention are given in (3.5). Equation (3.6) provides a formula for the sum of discounted interest payments under the half year convention. Before applying the formula in (3.6), the sum of discounted interest payments is restated as

$$\sum_{j=1}^{N}\frac{I_j}{(1+r_e)^j} = \frac{I_1}{1+r_e} + \sum_{j=2}^{N}\frac{I_j}{(1+r_e)^j}\,.$$

Then using (3.5) to replace I_1 and (3.6) with $v = 1/(1+r_e)$ produces

(1.6)

$$\begin{aligned}\sum_{j=1}^{N}\frac{I_j}{(1+r_e)^j} = (1-E)K\,\frac{\sqrt{1+r_d}-1}{1+r_e} \\ + \frac{(1-E)Kr_d\sqrt{1+r_d}}{(1+r_d)^N - 1}\Bigg[(1+r_d)^{N-1}\left(\frac{(1+\mathrm{r_e})^{N-1}-1}{r_e(1+\mathrm{r_e})^N}\right) \\ - \left(\frac{(1+\mathrm{r_e})^{N-1}-(1+\mathrm{r_d})^{N-1}}{(r_e-r_d)(1+\mathrm{r_e})^N}\right)\Bigg].\end{aligned}$$

Using (1.6) to replacing the sum of discounted interest payments in equation (1.5) and using (3.5) to replace P yields the CRF formula in (1.7).

(1.7)

$$CRF = \frac{r_e(1+r_e)^N}{(1-s)[(1+r_e)^N-1]}\left\{\frac{E}{\sqrt{1+r_e}} - s\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^j} - s(1-E)\frac{\sqrt{1+r_d}-1}{1+r_e}\right.$$
$$-\,s(1-E)\frac{r_d\sqrt{1+r_d}}{(1+r_d)^N-1}\left[(1+r_d)^{N-1}\left(\frac{(1+\mathrm{r_e})^{N-1}-1}{r_e(1+\mathrm{r_e})^N}\right)\right.$$
$$\left.\left.-\left(\frac{(1+\mathrm{r_e})^{N-1}-(1+\mathrm{r_d})^{N-1}}{(r_e-r_d)(1+\mathrm{r_e})^N}\right)\right] + (1-E)\left(\frac{r_d(1+r_d)^{N-1/2}}{(1+r_d)^N-1}\right)\left(\frac{(1+r_e)^N-1}{r_e(1+r_e)^N}\right)\right\}$$

Using the parameter values in Table 1-3, with a five year capital cost recovery period and straight line depreciation, equation (1.7) yields a CRF of 0.270743. With an initial capital investment of $1 million, the annual payment is $270,743. Table 1-5 shows the corresponding cash flow summary.

Table 1-5 Cash flow summary for 5 year, $1 million investment with half year convention and straight line depreciation

Capital Recovery Year	1	2	3	4	5
Revenue	$270,743	$270,743	$270,743	$270,743	$270,743
Depreciation	$200,000	$200,000	$200,000	$200,000	$200,000
Interest on debt	$17,204	$27,952	$21,656	$14,920	$7,712
Income Tax	$22,358	$17,870	$20,499	$23,312	$26,322
Return on equity (ROE)	$29,150	$47,838	$37,381	$25,984	$13,557
Revenue in excess of taxes, interest and ROE	$202,031	$177,083	$191,207	$206,527	$223,152
Repayment of debt principal	$100,685	$89,937	$96,233	$102,969	$110,177
Repayment of equity investment	$101,346	$87,146	$94,974	$103,558	$112,975
Debt Remaining	$399,315	$309,378	$213,145	$110,177	$0
Equity Remaining	$398,654	$311,508	$216,534	$112,975	$0

The calculation of the values in Table 1-5 is identical to the corresponding values in Table 1-4 except that the year 1 interest on the debt and the year 1 return on equity reflect a half year period. The interest on debt in year 1 is equal to the product of the initial debt and the half year interest rate $\sqrt{1+\mathrm{r}_d}-1$. The return on equity in year 1 is equal to the product of the equity investment and the half year rate of return $\sqrt{1+\mathrm{r}_e}-1$. The cash flow summary shows that the revenue payment determined by the CRF is at the necessary and sufficient level to provide for the income taxes associated with the revenue payments as well as the required return on and return of the capital investment.

Changing the depreciation assumption to 3 year MACRS produces a CRF of 0.254135. The MACRS depreciation factors are shown in Table 1-9. The lower CRF relative to the straight line depreciation example reflects the lower tax payment under MACRS due to the accelerated depreciation schedule. In years 1 and 2, the tax payment in Table 1-6 is negative due to the accelerated depreciation assumption.[7] The cash flow summary in Table 1-6 shows that the revenue payment determined by the CRF, using 3 year MACRS depreciation, is at the necessary and sufficient level to provide for the taxes associated with the revenue payment as well as the required return on and return of the capital investment.

Table 1-6 Cash flow summary for 5 year, $1 million investment with 3 year MACRS

Capital Recovery Year	1	2	3	4	5
Revenue	$254,135	$254,135	$254,135	$254,135	$254,135
Depreciation	$333,300	$444,500	$148,100	$74,100	$0
Interest on debt	$17,204	$27,952	$21,656	$14,920	$7,712
Income Tax	($40,244)	($91,169)	$35,237	$68,952	$102,906
Return on equity (ROE)	$29,150	$42,319	$20,108	$10,400	$3,572
Revenue in excess of taxes, interest and ROE	$248,025	$275,033	$177,134	$159,863	$139,945
Repayment of debt principal	$100,685	$89,937	$96,233	$102,969	$110,177
Repayment of equity investment	$147,340	$185,096	$80,902	$56,895	$29,768
Debt Remaining	$399,315	$309,378	$213,145	$110,177	$0
Equity Remaining	$352,660	$167,564	$86,663	$29,768	$0

Assuming 15 year MACRS depreciation results in a CRF of 0.360556. The corresponding cash flow summary is given in Table 1-7. The CRF is higher in this case, relative to the straight line and 3 year MACRS examples, because the income tax payments are higher. The depreciation offset is lower in each year and the investment is not fully depreciated during the capital recovery term.

7 It is assumed that the investor would use the negative tax liability from this project as an offset against the tax liability resulting from other revenue.

Table 1-7 Cash flow summary for 5 year, $1 million investment with 15 year MACRS

Capital Recovery Year	1	2	3	4	5
Revenue	$360,556	$360,556	$360,556	$360,556	$360,556
Depreciation	$50,000	$95,000	$85,500	$76,950	$69,255
Interest on debt	$17,204	$27,952	$21,656	$14,920	$7,712
Income Tax	$122,504	$99,223	$105,820	$112,203	$118,427
Return on equity (ROE)	$29,150	$49,078	$37,755	$25,864	$13,311
Revenue in excess of taxes, interest and ROE	$191,698	$184,302	$195,325	$207,569	$221,106
Repayment of debt principal	$100,685	$89,937	$96,233	$102,969	$110,177
Repayment of equity investment	$91,013	$94,365	$99,093	$104,600	$110,929
Debt Remaining	$399,315	$309,378	$213,145	$110,177	$0
Equity Remaining	$408,987	$314,622	$215,529	$110,929	$0

Assuming 100 percent bonus depreciation results in a CRF of 0.236550. The corresponding cash flow summary is given in Table 1-8.

Table 1-8 Cash flow summary for 5 year, $1 million investment with bonus depreciation

Capital Recovery Year	1	2	3	4	5
Revenue	$236,550	$236,550	$236,550	$236,550	$236,550
Depreciation	$1,000,000	$0	$0	$0	$0
Interest on debt	$17,204	$27,952	$21,656	$14,920	$7,712
Income Tax	($326,001)	$87,110	$89,739	$92,552	$95,562
Return on equity (ROE)	$29,150	$10,139	$7,569	$5,007	$2,475
Revenue in excess of taxes, interest and ROE	$516,197	$111,349	$117,585	$124,070	$130,800
Repayment of debt principal	$100,685	$89,937	$96,233	$102,969	$110,177
Repayment of equity investment	$415,512	$21,412	$21,352	$21,101	$20,623
Debt Remaining	$399,315	$309,378	$213,145	$110,177	$0
Equity Remaining	$84,488	$63,077	$41,725	$20,623	$0

In each example, the annual revenue payment, equal to the product of the capital investment and the CRF obtained from equation (1.7) is at the necessary and sufficient level to provide for the income taxes associated with the annual revenue payments, and the required return on and return of the capital investment.

Table 1-9 Modified Accelerated Cost Recovery System (MACRS) with half year convention[8]

Year	3 year Depreciation Factors	5 year Depreciation Factors	10 year Depreciation Factors	15 year Depreciation Factors	20 year Depreciation Factors
1	33.33%	20.00%	10.00%	5.00%	3.750%
2	44.45%	32.00%	18.00%	9.50%	7.219%
3	14.81%	19.20%	14.40%	8.55%	6.677%
4	7.41%	11.52%	11.52%	7.70%	6.177%
5		11.52%	9.22%	6.93%	5.713%
6		5.76%	7.37%	6.23%	5.285%
7			6.55%	5.90%	4.888%
8			6.55%	5.90%	4.522%
9			6.56%	5.91%	4.462%
10			6.55%	5.90%	4.461%
11			3.28%	5.91%	4.462%
12				5.90%	4.461%
13				5.91%	4.462%
14				5.90%	4.461%
15				5.91%	4.462%
16				2.95%	4.461%
17					4.462%
18					4.461%
19					4.462%
20					4.461%
21					2.231%

1.3 Hybrid Approach

The original CRFs calculated for PJM black service capital recovery used a hybrid approach with respect to the timing of cash flows. The cash flow to the equity investors and the corresponding present value calculations assumed the half year convention. The debt payments were treated as end of year payments. To derive a formula for the hybrid approach, the interest payments (I_j) and the debt payment (P) in the CRF formula in (1.5) are treated as end of year payments rather than midyear payments. An expression for the CRF under the hybrid approach is given by

8 See Appendix A, Table A-1, IRS Publication 946, United States Department of Treasury (2020).

(1.8)

$$c = \frac{r_e(1+r_e)^N}{(1-s)[(1+r_e)^N-1]}\left\{\frac{E}{\sqrt{1+r_e}} - s\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^j} - \frac{s}{K}\sum_{j=1}^{N}\frac{I_j}{(1+r_e)^j} + \frac{P}{K}\frac{(1+r_e)^N-1}{r_e(1+r_e)^N}\right\}$$

where the interest payments (I_j) and the debt payment (P) correspond to an end of year debt repayment approach. Equation (1.8) is identical to (1.5) except for difference in the definitions of the parameters I_j and P.

Formulas for the debt payment and interest portion of the debt payment, assuming a term of N years and end of year payments are given in (3.2). The sum of discounted interest payments, assuming end of year debt payments, is provided in (1.3). Replacing P in (1.8) using (3.2) and replacing the sum of discounted interest payments in (1.8) using (1.3) yields

(1.9)

$$\begin{aligned} c = \frac{r_e(1+r_e)^N}{(1-s)[(1+r_e)^N-1]}\Bigg\{&\frac{E}{\sqrt{1+r_e}} - s\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^j} \\ &- s(1-E)\frac{r_d}{(1+r_d)^N-1}\left[(1+r_d)^N\left(\frac{(1+r_e)^N-1}{r_e(1+r_e)^N}\right) - \frac{(1+r_e)^N-(1+r_d)^N}{(r_e-r_d)(1+r_e)^N}\right] \\ &+ (1-E)\frac{r_d(1+r_d)^N}{(1+r_d)^N-1}\left(\frac{(1+r_e)^N-1}{r_e(1+r_e)^N}\right)\Bigg\} \end{aligned}$$

Using the parameter values in Table 1-3, with a five year capital cost recovery period and straight line depreciation, equation (1.9) yields a CRF of 0.274194. With an initial capital investment of $1 million, the annual payment is $274,194. Table 1-10 shows the corresponding cash flow summary. The CRF and corresponding annual revenue is higher in comparison to the half year convention approach (see Table 1-5). This is due to the higher interest payments under the hybrid approach.

Table 1-10 Cash flow summary for 5 year, $1 million investment with hybrid approach and straight line depreciation

Capital Recovery Year	1	2	3	4	5
Revenue	$274,194	$274,194	$274,194	$274,194	$274,194
Depreciation	$200,000	$200,000	$200,000	$200,000	$200,000
Interest on debt	$35,000	$28,914	$22,402	$15,434	$7,978
Income Tax	$16,368	$18,909	$21,629	$24,539	$27,652
Return on equity (ROE)	$29,150	$47,192	$36,855	$25,603	$13,350
Revenue in excess of taxes, interest and ROE	$193,677	$179,179	$193,310	$208,620	$225,215
Repayment of debt principal	$86,945	$93,032	$99,544	$106,512	$113,968
Repayment of equity investment	$106,731	$86,148	$93,766	$102,108	$111,247
Debt Remaining	$413,055	$320,023	$220,479	$113,968	$0
Equity Remaining	$393,269	$307,121	$213,355	$111,247	$0

The cash flow summary reflects the hybrid approach. The debt entries in Table 1-10 reflect end of year interest and payments, and the return on equity calculations adhere to the half year convention. The rows for interest on debt, repayment of debt principal and debt remaining in Table 1-10 are identical to the corresponding rows in the Table 1-4 which assumes end of year cash flow for debt and equity.

Using the parameter values in Table 1-3, with a five year capital cost recovery period and 15 year MACRS depreciation, equation (1.9) yields a CRF of 0.364007. With an investment of $1 million, the annual payment is $364,007. Table 1-11 shows the corresponding cash flow summary.

Table 1-11 Cash flow summary for 5 year, $1 million investment with hybrid approach and 15 year MACRS depreciation

Capital Recovery Year	1	2	3	4	5
Revenue	$364,007	$364,007	$364,007	$364,007	$364,007
Depreciation	$50,000	$95,000	$85,500	$76,950	$69,255
Interest on debt	$35,000	$28,914	$22,402	$15,434	$7,978
Income Tax	$116,514	$100,263	$106,950	$113,430	$119,757
Return on equity (ROE)	$29,150	$48,432	$37,228	$25,482	$13,104
Revenue in excess of taxes, interest and ROE	$183,344	$186,398	$197,428	$209,662	$223,168
Repayment of debt principal	$86,945	$93,032	$99,544	$106,512	$113,968
Repayment of equity investment	$96,398	$93,367	$97,884	$103,150	$109,201
Debt Remaining	$413,055	$320,023	$220,479	$113,968	$0
Equity Remaining	$403,602	$310,235	$212,351	$109,201	$0

There has been debate regarding the calculation of the CRF for black start service capital recovery.[9] The original CRF values are shown in Table 1-12. The CRF were calculated using the hybrid approach and parameter assumptions in Table 1-3 with 15 year MACRS depreciation factors.

Table 1-12 CRF for capital investment to provide black start service PJM

Capital Recovery Term (years)	CRF
5	0.363
10	0.198
15	0.146
20	0.125

Formula (1.9) returns a CRF of 0.364007 for a CRF with a five year term under the same parameter and depreciation assumptions. The formula result does not match the value in Table 1-12 because the CRF values were originally calculated using an iterative solver which introduced some error. The CRF values were also truncated. Table 1-13 provides a comparison of the results of the iterative solver, prior to the truncation, and the CRF values produced using formula in (1.9).

Table 1-13 Comparison of CRF using the hybrid approach formula versus an iterative solver

Capital Recovery Term (years)	CRF Using Hybrid Formula	CRF using Iterative Solver	Absolute Error	Relative Error
5	0.364007	0.363956	-0.000052	-0.014%
10	0.198706	0.198694	-0.000012	-0.006%
15	0.146310	0.146312	0.000001	0.001%
20	0.125165	0.125171	0.000006	0.005%

9 See FERC Docket No. ER21-1635-000 and Docket No. EL21-91-000.

2 Updating the CRF to reflect a change in the tax law

A change in the income tax rate or the eligible depreciation offset can have a significant impact on the recovery of a capital investment. The Tax Cuts and Jobs Act (TCJA) of 2017 lowered the federal corporate income tax rate from 35 percent to 21 percent.[10] [11] The TCJA also introduced bonus depreciation that allows for increased depreciation during the first year of an asset's service life.[12]

When the tax law changes during the capital cost recovery period or the initial CRF is incorrect, it is necessary to update the CRF to avoid an incorrect capital cost recovery.[13] The updated CRF should adhere to the previously stated requirements that revenue payments resulting from the CRF provide for the return on and return of the capital investment in addition to covering the income tax liabilities associated with the revenue payments. The updated CRF will need to incorporate any lag between the change in the tax rate and the incorporation of the new tax rate in the revenue payment. The procedure for establishing the updated CRF can be described as a two step process (1) the outstanding capital investment is determined as of the effective date of the updated CRF and (2) an updated CRF is calculated based on the remaining service term and the outstanding capital investment amount.

2.1 *Updated CRF for the flow to equity (FTE) model*

The updated model incorporates a change in the tax rate and a change in the CRF. The two changes are treated independently in order to reflect the delayed implementation of a revised

[10] Tax Cuts and Jobs Act, Pub. L. No. 115-97, 131 Stat. 2096, Stat. 2105 (2017).

[11] 26 U.S. Code §11(b).

[12] Bonus depreciation is 100 percent for capital investments placed in service after September 27, 2017 and before January 1, 2023. Bonus depreciation is 80 percent for capital investments placed in service after December 31, 2022 and before January 1, 2024, and the bonus depreciation level is reduced by 20 percent for each subsequent year through 2026. Capital investments placed in service after December 31, 2026 are not eligible for bonus depreciation. See 26 U.S. Code §168(k)(6)(A).

[13] PJM black start resources that began capital cost recovery between January 1, 2018 and June 6, 2021 were assigned an incorrect CRF based on outdated and incorrect income tax rules.

CRF. Variable m represents the capital recovery year during which the tax change occurs, and variable q represents the first capital recovery year during which the updated CRF is effective. Variable γ $(0 \leq \gamma < 1)$ is the fractional portion of year m for which the old tax rate is applicable. Variable μ $(0 \leq \mu < 1)$ is the fractional portion of year q for which the old CRF is applicable. For example, consider a black start unit that began service on December 1, 2017 and assume the updated CRF will be effective on November 1, 2021. The TCJA income tax rate change was effective on January 1, 2018 which was one month into the service term, so that m is 1 and γ is 0.0833. The updated CRF became effective eleven months into service year 4, so that q is 4 and μ is 0.9167. Table 2-1 provides a description of the model variables. The initial effective income tax rate is s_1 and the initial CRF is c_1. In year m the effective income tax rate changes to s_2, and in year q the CRF is updated to c_2.

Table 2-1 Parameter descriptions for the updated CRF for the FTE model

Parameter	Description
E	Equity funding percent
r_e	Rate of return on equity
r_d	Debt interest rate
s_1	Initial effective income tax rate
s_2	Updated effective income tax rate
c_1	Initial CRF
c_2	Updated CRF
N	Capital investment recovery period
m	Recovery year during which the income tax rate is updated
γ	Portion of recovery year m for which tax rate s_1 applies
q	Recovery year during which the CRF is updated
μ	Portion of recovery year q for which CRF c_1 applies
δ_j	Depreciation factor for year j

The updated CRF is given by equation (2.1).

(2.1)

$$c_2 = \frac{r_e(1+r_e)^{N-q}}{(1-s_2)[(1+r_e)^{N-q+1} - 1 - \mu r_e(1+r_e)^{N-q}]}\left\{F_A(1+r_e) - \mu c_1(1-s_2) - s_2\sum_{j=q}^{N}\frac{\delta_j}{(1+r_e)^{j-q}}\right.$$
$$- (1-E)s_2\frac{r_d(1+r_d)^{q-3/2}}{(1+r_d)^N - 1}\left\{(1+r_d)^{N-q+1}\left(\frac{(1+r_e)^{N-q+1}-1}{r_e(1+r_e)^{N-q}}\right)\right.$$
$$\left. - \frac{(1+r_e)^{N-q+1} - (1+r_d)^{N-q+1}}{(r_e - r_d)(1+r_e)^{N-q}}\right\}$$
$$\left. + (1-E)\left(\frac{r_d(1+r_d)^{N-1/2}}{(1+r_d)^N - 1}\right)\left(\frac{(1+r_e)^{N-q+1}-1}{r_e(1+r_e)^{N-q}}\right)\right\}$$

The factor F_A, when multiplied by the initial capital investment K_0, is the remaining equity investment at the midpoint of year $q-1$ for the case where $m < q$.[14] The formula for F_A is given in (2.2).[15]

(2.2)

$$F_A = F_{m-1}(1+r_e)^{\alpha} - c_1(1-s_2)\frac{(1+r_e)^{q-m}-1}{r_e} - s_2X + \gamma(c_1 - \delta_m)(s_1 - s_2)(1+\mathrm{r_e})^{q-m-1}$$
$$- (1-E)[\gamma s_1 + (1-\gamma)s_2]Z(1+r_e)^{q-m-1}$$
$$- (1-E)s_2\frac{r_d(1+r_d)^{m-1/2}}{(1+r_d)^N - 1}\left[(1+r_d)^{N-m}\left(\frac{(1+r_e)^{q-m-1}-1}{r_e}\right)\right.$$
$$\left. - \frac{(1+r_e)^{q-m-1} - (1+r_d)^{q-m-1}}{r_e - r_d}\right] + (1-E)\left(\frac{r_d(1+r_d)^{N-1/2}}{(1+r_d)^N - 1}\right)\left(\frac{(1+r_e)^{q-m}-1}{r_e}\right)$$

$$\alpha = \begin{cases} q - 3/2, & m = 1 \\ q - m, & m > 1 \end{cases}$$

14 In the case that the CRF is updated in the same year as the tax law change (i.e. $m = q$), this interpretation of factor F_A does not hold because it includes a portion of the taxes paid in year q in addition to the remaining equity investment at the midpoint of year $q-1$.

15 Factor F_A was identified as F_{q-1} in IMM comments (November 11, 2021) and in IMM errata filing (November 18, 2021) in Docket EL21-91.

$$X = \begin{cases} 0 & , \quad m = q \\ \sum_{j=m}^{q-1} \delta_j (1 + r_e)^{q-j-1}, & m < q \end{cases}$$

$$Z = \begin{cases} \sqrt{1 + r_d} - 1 & , \quad m = 1 \\ \frac{r_d (1 + r_d)^{m-1}}{\sqrt{1 + r_d}} \left(\frac{(1 + r_d)^{N-m+1} - 1}{(1 + r_d)^N - 1} \right), & m > 1 \end{cases}$$

If $m > 1$ the factor F_{m-1} when multiplied by the initial capital investment K_0 gives the remaining equity investment at the midpoint of service year $m - 1$. Factor F_{m-1}, for the case $m > 1$ is given in equation (2.3). In the case that the tax law change occurs in the first capital recovery year ($m = 1$), factor F_{m-1} is defined to be the equity funding percent (i.e. $F_0 = E$).

(2.3)

$$\begin{aligned} F_{m-1} = E(1 + r_e)^{m-3/2} - \mathrm{c}_1(1 - s_1) \frac{(1 + r_e)^{m-1} - 1}{r_e} - \mathrm{s}_1 \sum_{j=1}^{m-1} \delta_j (1 + r_e)^{m-1-j} \\ - (1 - E)\mathrm{s}_1 \left(\sqrt{1 + r_d} - 1 \right) (1 + r_e)^{m-2} \\ - \frac{(1 - E)\mathrm{s}_1 r_d \sqrt{1 + r_d}}{(1 + r_d)^N - 1} \left[(1 + r_d)^{N-1} \frac{(1 + r_e)^{m-2} - 1}{r_e} - \frac{(1 + r_e)^{m-2} - (1 + r_d)^{m-2}}{r_e - r_d} \right] \\ + (1 - E) \left(\frac{r_d (1 + r_d)^{N-1/2}}{(1 + r_d)^N - 1} \right) \left(\frac{(1 + r_e)^{m-1} - 1}{r_e} \right). \end{aligned}$$

As an example, consider a $1 million capital investment with a five year capital cost recovery period that began on May 1, 2016 and assume the updated CRF has an effective date of January 1, 2019. The cost recovery period began prior to the TCJA. Assuming the initial CRF was calculated using the parameter values in Table 1-3 and 15 year MACRS depreciation, equation (1.7) returns a CRF of 0.360556. The corresponding annual revenue payment is $360,556. The tax rate change has an effective date of January 1, 2018 which is eight months into the second capital cost recovery year ($m = 2, \gamma = 0.6667$). Table 2-2 shows the result if the CRF is not updated to reflect the change in the federal income tax rate.

Table 2-2 Cash flow summary with no update to the CRF under FTE, $1 million investment

Capital Recovery Year	1	2	3	4	5
Revenue	$360,556	$360,556	$360,556	$360,556	$360,556
Depreciation	$50,000	$95,000	$85,500	$76,950	$69,255
Interest on debt	$17,204	$27,952	$21,656	$14,920	$7,712
Income Tax	$122,504	$88,412	$71,231	$75,528	$79,717
Return on equity (ROE)	$29,150	$49,078	$36,457	$20,260	$2,634
Revenue in excess of taxes, interest and ROE	$191,698	$195,113	$231,212	$249,848	$270,493
Repayment of debt principal	$100,685	$89,937	$96,233	$102,969	$110,177
Repayment of equity investment	$91,013	$105,176	$134,979	$146,880	$21,952
Debt Remaining	$399,315	$309,378	$213,145	$110,177	$0
Equity Remaining	$408,987	$303,811	$168,832	$21,952	$0
Excess Revenue to equity investors	$0	$0	$0	$0	$138,364

The revenue remains at the annual rate of $360,556 for the duration of the capital recovery period. The first year cash flow is identical to Table 1-7, which was based on the same initial assumptions. For years 2 through 5 , the cash flow in Table 2-2 reflects the lower federal income tax rate. The repayment of the equity investment in years 2 through 4 is higher in comparison to Table 1-7 and the remaining equity is reduced to $21,952 in year 4. In year 5, the remaining equity is repaid and the equity investors receive an additional $138,364. The equity investor's rate of return, intended to be 12.0 percent, is 19.6 percent. Table 2-2 shows that the failure to update the CRF will lead to over recovery for the equity investor.

Table 2-3 shows the cash flow that results when the CRF is updated. The effective date of the updated CRF is January 1, 2019 ($q = 3, \mu = 0.6667$). Using (2.1) the updated CRF value is 0.284726 and the updated annual revenue amount is $284,726. In recovery year 3, the annual revenue rate is reduced to $284,726 and there is no over recovery by the equity investors.[16] The equity investor's rate of return is 12.0 percent.

16 The revenue for year 3 reflects eight months at the original CRF and four months at the updated CRF.

Table 2-3 Cash flow summary with an updated CRF under FTE, $1 million investment

Capital Recovery Year	1	2	3	4	5
Revenue	$360,556	$360,556	$335,279	$284,726	$284,726
Depreciation	$50,000	$95,000	$85,500	$76,950	$69,255
Interest on debt	$17,204	$27,952	$21,656	$14,920	$7,712
Income Tax	$122,504	$88,412	$64,125	$54,212	$58,401
Return on equity (ROE)	$29,150	$49,078	$36,457	$22,440	$11,618
Revenue in excess of taxes, interest and ROE	$191,698	$195,113	$213,040	$193,154	$206,995
Repayment of debt principal	$100,685	$89,937	$96,233	$102,969	$110,177
Repayment of equity investment	$91,013	$105,176	$116,808	$90,185	$96,818
Debt Remaining	$399,315	$309,378	$213,145	$110,177	$0
Equity Remaining	$408,987	$303,811	$187,003	$96,818	$0
Excess Revenue to equity investors	$0	$0	$0	$0	$0

Table 2-3 shows that the CRF determined by equation (2.1) provides the necessary and sufficient revenue amount to cover the income tax liability and the return on and return of the investment capital.

2.2 Generalized Formula

The CRF formula given in (2.1) is a generalization or extension of the FTE CRF formula, under the half year convention, in (1.7). The extended formula in (2.1) reduces to the formula in (1.7) under the appropriate parameter settings. Formula (1.7) returns a CRF value for a new investment that is just beginning the capital recovery period. To get an initial CRF value from (2.1) the income tax rate change is assumed to have occurred on or before the investment was placed into service and the CRF update is assumed to have occurred at the beginning of the capital recovery term. The corresponding settings for the parameters are $m = 1$, $q = 1$, $\gamma = 0$, and $\mu = 0$. Under these assumptions, (2.1) reduces to

$$c_2 = \frac{r_e(1+r_e)^{N-1}}{(1-s_2)[(1+r_e)^N-1]}\left\{E\sqrt{1+r_e} - (1-E)s_2\left[\left(\sqrt{1+r_d}-1\right)+\frac{r_d}{\sqrt{1+r_d}}\right] - s_2\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^{j-1}}\right.$$
$$- (1-E)s_2\frac{r_d(1+r_d)^{-1/2}}{(1+r_d)^N-1}\left\{(1+r_d)^N\left(\frac{(1+r_e)^N-1}{r_e(1+r_e)^{N-1}}\right) - \frac{(1+r_e)^N-(1+r_d)^N}{(r_e-r_d)(1+r_e)^{N-1}}\right\}$$
$$\left. + (1-E)\left(\frac{r_d(1+r_d)^{N-1/2}}{(1+r_d)^N-1}\right)\left(\frac{(1+r_e)^N-1}{r_e(1+r_e)^{N-1}}\right)\right\}.$$

Using (3.7)

$$\frac{(1+\mathrm{r_e})^N-1}{r_e} = (1+\mathrm{r_e})^{N-1} + \frac{(1+\mathrm{r_e})^{N-1}-1}{r_e}$$

and using (3.8)

$$\frac{(1+\mathrm{r_e})^N-(1+\mathrm{r}_d)^N}{r_e-r_d} = (1+\mathrm{r_e})^{N-1} + (1+r_d)\frac{(1+\mathrm{r_e})^{N-1}-(1+\mathrm{r_d})^{N-1}}{r_e-r_d} .$$

After making the substitutions, the expression for c_2 can be restated as

$$c_2 = \left\{\frac{r_e(1+r_e)^{N-1}}{(1-s_2)[(1+r_e)^N-1]}\right\}\left\{E\sqrt{1+r_e} - (1-E)s_2\left(\sqrt{1+r_d}-1\right) - s_2\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^{j-1}}\right.$$
$$- (1-E)s_2\frac{r_d(1+r_d)^{1/2}}{(1+r_d)^N-1}\left[(1+r_d)^{N-1}\frac{(1+\mathrm{r_e})^{N-1}-1}{r_e(1+\mathrm{r_e})^{N-1}}\right.$$
$$\left.\left. - \frac{(1+\mathrm{r_e})^{N-1}-(1+\mathrm{r_d})^{N-1}}{(r_e-r_d)(1+\mathrm{r_e})^{N-1}}\right] + (1-E)\left(\frac{r_d(1+r_d)^{N-1/2}}{(1+r_d)^N-1}\right)\left(\frac{(1+r_e)^N-1}{r_e(1+r_e)^{N-1}}\right)\right\}.$$

The formula in (1.7) is obtained from the expression above by multiplying the first bracketed term by $(1+r_e)$ and the second bracketed term by $1/(1+r_e)$.

2.3 Derivation of the updated CRF under the FTE model

The formulas in (2.1)-(2.3) are derived in a series of cases where assumptions are made regarding the timing of the change in the income tax rules and when the CRF is updated to reflect this change. The derivations for all cases will follow the same process. The outstanding equity investment at the midpoint of the capital recovery year during which the CRF is updated is determined. This value is denoted by $K_q^{(1,e)}$. The updated CRF (c_2) is determined by requiring that the present value of the after tax cash flow resulting from revenue payments after the CRF is are equal $K_q^{(1,e)}$. The first case assumes the income tax rule change occurred sometime after the first capital recovery year ($m > 1$) and the CRF is updated in a future year ($m < q$). Working through the cases, the timing assumptions are relaxed and it is shown that equations (2.1) - (2.3) hold for $1 \leq m \leq q \leq N$.

2.3.1 Case I: Tax rate change occurs in year 2 or later and the CRF is updated in the following year or later ($1 < m < q \leq N$)

In capital recovery year 1, the revenue is $c_1 K$ and the income tax payment is $(c_1 K_0 - \delta_1 K_0 - I_1)s_1$. The repayment to the equity investor at the midpoint of year 1 is the revenue net of taxes, the debt payment and return on equity and is given by

$$c_1 K_0(1 - s_1) + \delta_1 K_0 s_1 + I_1 s_1 - P - EK_0\left(\sqrt{1 + r_e} - 1\right).$$

EK_0, the product of the equity funding percent and the total capital investment, is the equity investment at the start of the capital recovery period. I_1 is the interest portion of the debt payment P in year 1 and the return on equity, reflecting the half year convention, is given by $EK_0\left(\sqrt{1 + r_e} - 1\right)$. The equity investment remaining at the midpoint of year 1 ($K_1^{(e)}$), equal to the initial equity investment less the revenue left over after paying the income tax, debt payment and investment return, is

$$\begin{aligned} K_1^{(e)} &= EK_0 - \left(\mathrm{c}_1 \mathrm{K}_0(1 - s_1) + \delta_1 \mathrm{K}_0 \mathrm{s}_1 + I_1 \mathrm{s}_1 - \mathrm{P} - \mathrm{EK}_0\left(\sqrt{1 + r_e} - 1\right)\right) \\ &= EK_0\sqrt{1 + r_e} - \mathrm{c}_1 \mathrm{K}_0(1 - s_1) - \delta_1 \mathrm{K}_0 \mathrm{s}_1 - I_1 \mathrm{s}_1 + P. \end{aligned}$$

The revenue net of taxes, the debt payment and return on equity for year 2 is

$$\mathrm{c}_1 \mathrm{K}_0(1 - s_1) + \delta_2 \mathrm{K}_0 \mathrm{s}_1 + I_2 \mathrm{s}_1 - \mathrm{P} - K_1^{(e)} r_e$$

and the equity investment remaining at the midpoint of year 2 is

$$\begin{aligned} K_2^{(e)} &= K_1^{(e)} - \left(\mathrm{c}_1 \mathrm{K}_0(1 - s_1) + \delta_2 \mathrm{K}_0 \mathrm{s}_1 + I_2 \mathrm{s}_1 - \mathrm{P} - K_1^{(e)} r_e\right) \\ &= K_1^{(e)}(1 + r_e) - \mathrm{c}_1 \mathrm{K}_0(1 - s_1) - \delta_2 \mathrm{K}_0 \mathrm{s}_1 - I_2 \mathrm{s}_1 + P \\ &= EK_0(1 + r_e)^{3/2} - \mathrm{c}_1 \mathrm{K}_0(1 - s_1)[(1 + r_e) + 1] - \mathrm{K}_0 \mathrm{s}_1[\delta_1(1 + r_e) + \delta_2] - \mathrm{s}_1[I_1(1 + r_e) + I_2] \\ &\quad + P[(1 + r_e) + 1]. \end{aligned}$$

Proceeding in this manner up to the year before the tax change, the equity investment remaining at the midpoint of year $m-1$ is

$$K_{m-1}^{(e)} = EK_0(1+r_e)^{m-3/2} - \mathrm{c}_1\mathrm{K}_0(1-s_1)\sum_{j=0}^{m-2}(1+r_e)^j - \mathrm{K}_0\mathrm{s}_1\sum_{j=1}^{m-1}\delta_j(1+r_e)^{m-1-j}$$
$$- \mathrm{s}_1\sum_{j=1}^{m-1}I_j(1+r_e)^{m-1-j} + P\sum_{j=1}^{m-1}(1+r_e)^{j-1}\,.$$

Using equation (3.1)

(2.4)

$$\sum_{j=0}^{m-2}(1+r_e)^j = \frac{(1+r_e)^{m-1}-1}{r_e}.$$

The sum of the interest payments can be restated as

$$\sum_{j=1}^{m-1}I_j(1+r_e)^{m-1-j} = I_1(1+r_e)^{m-2} + (1+r_e)^{m-1}\sum_{j=2}^{m-1}I_j\left(\frac{1}{1+r_e}\right)^j$$

and then using (3.5) to replace the I_1 and (3.6) with $D=(1-E)K_0$, $H=2$, $W=m-1$ and $v=1/1+r_e$, the sum of interest payments is

(2.5)

$$\sum_{j=1}^{m-1}I_j(1+r_e)^{m-1-j}$$
$$= I_1(1+r_e)^{m-2}$$
$$+\frac{(1-E)K_0r_d\sqrt{1+r_d}}{(1+r_d)^N-1}\left[(1+r_d)^{N-1}\frac{(1+r_e)^{m-2}-1}{r_e}-\frac{(1+r_e)^{m-2}-(1+r_d)^{m-2}}{r_e-r_d}\right].$$

Making the substitutions allowed by (2.4) and (2.5) and using (3.5) to replace I_1 and P

$$
\begin{aligned}
K_{m-1}^{(e)} = K_0 \Bigg\{ & E(1+r_e)^{m-3/2} - \mathrm{c}_1(1-s_1)\frac{(1+r_e)^{m-1}-1}{r_e} - \mathrm{s}_1\sum_{j=1}^{m-1}\delta_j(1+r_e)^{m-1-j} \\
& -(1-E)\mathrm{s}_1\left(\sqrt{1+r_d}-1\right)(1+r_e)^{m-2} \\
& -(1-E)\mathrm{s}_1\frac{r_d\sqrt{1+r_d}}{(1+r_d)^N-1}\Bigg[(1+r_d)^{N-1}\frac{(1+r_e)^{m-2}-1}{r_e} \\
& -\frac{(1+r_e)^{m-2}-(1+r_d)^{m-2}}{r_e-r_d}\Bigg] + (1-E)\left(\frac{r_d(1+r_d)^{N-1/2}}{(1+r_d)^N-1}\right)\left(\frac{(1+r_e)^{m-1}-1}{r_e}\right)\Bigg\}.
\end{aligned}
$$

The bracketed term matches the expression for F_{m-1} in (2.3) and therefore

$$K_{m-1}^{(e)} = K_0 F_{m-1}\,.$$

The tax rate changes from s_1 to s_2 in year m. The revenue in year m will remain at $c_1 K_0$ since the CRF will not be changed until a later year under the current assumption that $m < q$. The tax rate is s_1 for the partial year γ and the tax rate is s_2 for the remaining $1-\gamma$ portion of the year. The tax payment for year m is

$$\gamma(c_1K_0 - \delta_m K_0 - I_m)s_1 + (1-\gamma)(c_1K_0 - \delta_m K_0 - I_m)s_2$$

or equivalently

$$c_1K_0s_2 - \delta_m K_0 s_2 + \gamma(c_1-\delta_m)K_0(s_1-s_2) - [\gamma s_1 + (1-\gamma)s_2]I_m.$$

The revenue net of taxes, the debt payment and return on equity for year m is

$$c_1K_0(1-s_2) + \delta_m K_0 s_2 - \gamma(c_1-\delta_m)K_0(s_1-s_2) + [\gamma s_1 + (1-\gamma)s_2]I_m - P - K_{m-1}^{(e)}r_e$$

and the equity investment remaining at the midpoint of year m, equal to $K_{m-1}^{(e)}$ reduced by the year m revenue net of taxes, the debt payment and return on equity, is

$$K_m^{(e)} = K_{m-1}^{(e)}(1+r_e) - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2) - \delta_m K_0 s_2 + \gamma(c_1-\delta_m)K_0(s_1-s_2) - [\gamma s_1 + (1-\gamma)s_2]I_m + P.$$

The revenue net of taxes, the debt payment and return on equity for year $m+1$, in the case that $q > m+1$, is

$$c_1K_0(1-s_2) + \delta_{m+1}K_0 s_2 + I_{m+1}s_2 - P - K_m^{(e)}r_e$$

and the equity investment remaining at the midpoint of year $m+1$ is

$$K_{m+1}^{(e)} = K_m^{(e)}(1 + \mathrm{r_e}) - \mathrm{c_1 K_0}(1 - \mathrm{s_2}) - \delta_{\mathrm{m+1}}\mathrm{K_0 s_2} - I_{m+1}s_2 + P\,.$$

After replacing $K_m^{(e)}$,

$$\begin{aligned} K_{m+1}^{(e)} = K_{m-1}^{(e)}(1 + \mathrm{r_e})^2 &- \mathrm{c_1 K_0}(1 - \mathrm{s_2})[(1 + r_e) + 1] - K_0 s_2[\delta_m(1 + \mathrm{r_e}) + \delta_{\mathrm{m+1}}] \\ &+ \gamma(c_1 - \delta_m)K_0(s_1 - s_2)(1 + \mathrm{r_e}) - [\gamma s_1 + (1-\gamma)s_2]I_m(1 + \mathrm{r_e}) - s_2 I_{m+1} \\ &+ P[(1 + r_e) + 1] \end{aligned}$$

For year $m + 2$, the revenue net of taxes, the debt payment and return on equity, in the case that $q > m + 2$, is

$$c_1 K_0(1 - s_2) + \delta_{m+2}K_0 s_2 + I_{m+2}s_2 - P - K_{m+1}^{(e)} r_e$$

and the capital investment remaining at the midpoint of year $m + 2$ is

$$K_{m+2}^{(e)} = K_{m+1}^{(e)}(1 + \mathrm{r_e}) - \mathrm{c_1 K_0}(1 - \mathrm{s_2}) - \delta_{\mathrm{m+2}}\mathrm{K_0 s_2} - I_{m+2}s_2 + P\,.$$

After replacing $K_{m+1}^{(e)}$,

$$\begin{aligned} K_{m+2}^{(e)} = K_{m-1}^{(e)}(1 + \mathrm{r_e})^3 &- \mathrm{c_1 K_0}(1 - \mathrm{s_2})[(1 + r_e)^2 + (1 + r_e) + 1] \\ &- K_0 s_2[\delta_m(1 + \mathrm{r_e})^2 + \delta_{m+1}(1 + \mathrm{r_e}) + \delta_{\mathrm{m+2}}] + \gamma(c_1 - \delta_m)K_0(s_1 - s_2)(1 + \mathrm{r_e})^2 \\ &- [\gamma s_1 + (1-\gamma)s_2]I_m(1 + \mathrm{r_e})^2 - s_2[I_{m+1}(1 + \mathrm{r_e}) + I_{m+2}] \\ &+ P[(1 + r_e)^2 + (1 + r_e) + 1] \end{aligned}$$

Noting the pattern,

$$\begin{aligned} K_{q-1}^{(e)} = K_{m-1}^{(e)}(1 + \mathrm{r_e})^{q-m} &- \mathrm{c_1 K_0}(1 - \mathrm{s_2}) \sum_{j=0}^{q-m-1} (1 + \mathrm{r_e})^j - K_0 s_2 \sum_{j=m}^{q-1} \delta_j (1 + \mathrm{r_e})^{q-1-j} \\ &+ \gamma(c_1 - \delta_m)K_0(s_1 - s_2)(1 + \mathrm{r_e})^{q-m-1} - [\gamma s_1 + (1-\gamma)s_2]I_m(1 + \mathrm{r_e})^{q-m-1} \\ &- s_2 \sum_{j=m+1}^{q-1} I_j(1 + \mathrm{r_e})^{q-1-j} + P \sum_{j=0}^{q-m-1} (1 + \mathrm{r_e})^j\,. \end{aligned}$$[17]

Using (3.1)

$$\sum_{j=0}^{q-m-1} (1 + \mathrm{r_e})^j = \frac{(1 + \mathrm{r_e})^{q-m} - 1}{\mathrm{r_e}}.$$

[17] This expression follows by first recognizing $q - 1 = m + (q - m - 1)$.

Rewriting the sum of compounded interest payments as

$$\sum_{j=m+1}^{q-1} I_j (1+\mathrm{r_e})^{q-1-j} = (1+\mathrm{r_e})^{q-1} \sum_{j=m+1}^{q-1} I_j \left(\frac{1}{1+\mathrm{r_e}}\right)^j$$

and then using (3.6) yields

(2.6)

$$\sum_{j=m+1}^{q-1} I_j (1+\mathrm{r_e})^{q-1-j}$$
$$= (1-E)K_0 \frac{r_d(1+r_d)^{m-1/2}}{(1+r_d)^N - 1} \left\{ (1+r_d)^{N-m} \left(\frac{(1+r_e)^{q-m-1} - 1}{r_e} \right) - \left(\frac{(1+r_e)^{q-m-1} - (1+r_d)^{q-m-1}}{r_e - r_d} \right) \right\}.$$

The values of P and I_m are given in (3.5).

Making the substitutions identified above and also replacing $K_{m-1}^{(e)}$ with $K_0 F_{m-1}$ yields

$$\begin{aligned}
K_{q-1}^{(e)} = K_0 \Bigg\{ & F_{m-1}(1+\mathrm{r_e})^{q-m} - \mathrm{c_1}(1-\mathrm{s_2}) \frac{(1+\mathrm{r_e})^{q-m} - 1}{\mathrm{r_e}} - s_2 \sum_{j=m}^{q-1} \delta_j (1+\mathrm{r_e})^{q-1-j} \\
& + \gamma(c_1 - \delta_m)(s_1 - s_2)(1+\mathrm{r_e})^{q-m-1} \\
& - (1-E)[\gamma s_1 + (1-\gamma)s_2] \frac{r_d(1+r_d)^{m-1}}{\sqrt{1+r_d}} \left(\frac{(1+r_d)^{N-m+1} - 1}{(1+r_d)^N - 1} \right) (1+\mathrm{r_e})^{q-m-1} \\
& - s_2(1-E) \frac{r_d(1+r_d)^{m-1/2}}{(1+r_d)^N - 1} \left\{ (1+r_d)^{N-m} \left(\frac{(1+r_e)^{q-m-1} - 1}{r_e} \right) \right. \\
& \left. - \left(\frac{(1+r_e)^{q-m-1} - (1+r_d)^{q-m-1}}{r_e - r_d} \right) \right\} \\
& + (1-E) \left(\frac{r_d(1+r_d)^{N-1/2}}{(1+r_d)^N - 1} \right) \left(\frac{(1+\mathrm{r_e})^{q-m} - 1}{\mathrm{r_e}} \right) \Bigg\}.
\end{aligned}$$

Note that the bracketed expression matches the definition of F_A in (2.2) so that in the case that $1 < m < q$,

(2.7)

$$K_{q-1}^{(e)} = K_0 F_A \,.$$

The CRF is updated in year q. The CRF c_1 determines the revenue for the partial year μ and the updated CRF c_2 determines the revenue for the remaining $1-\mu$ portion of the year. The revenue prior to the CRF update, net of taxes, the debt payment and the return on equity for year q is

$$\mu\left[c_1 K_0 - \left(c_1 K_0 - \delta_q K_0 - I_q\right)s_2 - P\right] - r_e K_{q-1}^{(e)}.$$

The equity investment remaining at the midpoint of service year q, after accounting for all revenue payments under the original CRF c_1 is

(2.8)

$$K_q^{(e,1)} = K_{q-1}^{(e)}(1+r_e) - \mu c_1 K_0(1-s_2) - \mu\left(\delta_q K_o s_2 + I_q s_2 - P\,\right)$$

and then using (2.7)

(2.9)

$$K_q^{(e,1)} = K_0 F_A(1+r_e) - \mu c_1 K_0(1-s_2) - \mu\left(\delta_q K_o s_2 + I_q s_2 - P\,\right).$$

$K_q^{(e,1)}$ represents the remaining equity capital at the midpoint of year q after accounting for revenue under the original CRF c_1.[18] An updated CRF c_2 is determined by requiring the present value of the future after tax cash flows to the equity investor is equal to $K_q^{(e,1)}$. The revenue after the CRF update in year q is $(1-\mu)c_2 K_0$ and the after tax cash flow to the equity investors is the revenue less income taxes and the debt payment,

$$(1-\mu)\left[c_2 K_0(1-s_2) + \delta_q K_0 s_2 + I_q s_2 - P\right].$$

The after tax cash flows for years $j = q+1$ through N are

$$c_2 K_0(1-s_2) + \delta_j K_0 s_2 + I_j s_2 - P\,.$$

---

[18] The superscript in the symbol, $K_q^{(e,1)}$, is used to recognize that this value does not represent the total equity investment remaining at the midpoint of year q. The after tax revenue attributable to the updated CRF c_2 in year q is not included.

The present value of the after tax cash flows, corresponding to revenue payments after the CRF update, is

(2.10)

$$(1-\mu)\left[c_2K_0(1-s_2)+\delta_qK_0s_2+I_qs_2-P\right]+\sum_{j=q+1}^{N}\frac{c_2K_0(1-s_2)+\delta_jK_0s_2+I_js_2-P}{(1+r_e)^{j-q}}.$$

Equation (2.10) can be restated as

$$c_2K_0(1-s_2)\sum_{j=q}^{N}\frac{1}{(1+r_e)^{j-q}}+K_0s_2\sum_{j=q}^{N}\frac{\delta_j}{(1+r_e)^{j-q}}+s_2\sum_{j=q}^{N}\frac{I_j}{(1+r_e)^{j-q}}-P\sum_{j=q}^{N}\frac{1}{(1+r_e)^{j-q}}$$
$$-\mu\left[c_2K_0(1-s_2)+\delta_qK_0s_2+I_qs_2-P\right].$$

Using (3.1)

$$\sum_{j=q}^{N}\frac{1}{(1+r_e)^{j-q}}=\frac{(1+r_e)^{N-q+1}-1}{r_e(1+r_e)^{N-q}}$$

and using (3.6)

$$\sum_{j=q}^{N}\frac{I_j}{(1+r_e)^{j-q}}=(1-E)K_0\frac{r_d(1+r_d)^{q-3/2}}{(1+r_d)^N-1}\left\{(1+r_d)^{N-q+1}\left(\frac{(1+r_e)^{N-q+1}-1}{r_e(1+r_e)^{N-q}}\right)\right.$$
$$\left.-\frac{(1+r_e)^{N-q+1}-(1+r_d)^{N-q+1}}{(r_e-r_d)(1+r_e)^{N-q}}\right\}.$$

Upon making these substitutions, the present value of the after tax cash flows is

(2.11)

$$c_2K_0(1-s_2)\left(\frac{(1+r_e)^{N-q+1}-1}{r_e(1+r_e)^{N-q}}\right)+K_0s_2\sum_{j=q}^{N}\frac{\delta_j}{(1+r_e)^{j-q}}$$
$$+s_2(1-E)K_0\frac{r_d(1+r_d)^{q-3/2}}{(1+r_d)^N-1}\left\{(1+r_d)^{N-q+1}\left(\frac{(1+r_e)^{N-q+1}-1}{r_e(1+r_e)^{N-q}}\right)\right.$$
$$\left.-\frac{(1+r_e)^{N-q+1}-(1+r_d)^{N-q+1}}{(r_e-r_d)(1+r_e)^{N-q}}\right\}-P\left(\frac{(1+r_e)^{N-q+1}-1}{r_e(1+r_e)^{N-q}}\right)$$
$$-\mu\left[c_2K_0(1-s_2)+\delta_qK_0s_2+I_qs_2-P\right]$$

Note that if $q=N$, (2.11) reduces to the after tax cash flow, after the CRF update, in year N

and therefore (2.11) holds for $q \leq N$.

Setting the expression for the after tax cash flows in (2.11) equal to $K_q^{(e,1)}$ and solving for c_2 yields

(2.12)

$$c_2 = \frac{r_e(1+r_e)^{N-q}}{(1-s_2)[(1+r_e)^{N-q+1} - 1 - \mu r_e(1+r_e)^{N-q}]} \left\{ \frac{K_q^{(e,1)}}{K_0} - s_2 \sum_{j=q}^{N} \frac{\delta_j}{(1+r_e)^{j-q}} \right.$$
$$- s_2(1-E)\frac{r_d(1+r_d)^{q-3/2}}{(1+r_d)^N - 1}\left\{(1+r_d)^{N-q+1}\left(\frac{(1+r_e)^{N-q+1}-1}{r_e(1+r_e)^{N-q}}\right)\right.$$
$$\left.\left. - \frac{(1+r_e)^{N-q+1} - (1+r_d)^{N-q+1}}{(r_e - r_d)(1+r_e)^{N-q}}\right\} + \frac{P}{K_0}\frac{(1+r_e)^{N-q+1}-1}{r_e(1+r_e)^{N-q}} + \mu\left[\delta_q s_2 + \frac{I_q}{K_0}s_2 - \frac{P}{K_0}\right]\right\}.$$

The expression for c_2 in (2.1) is obtained from (2.12) by using (2.9) to replace $K_q^{(e,1)}$ and (3.5) to replace P.

2.3.2 Case II: Tax rate change and CRF update occur in same year, after the first year ($1 < m = q \leq N$)

In Case II it is assumed the change in the income tax rate and the CRF update to reflect the income tax change occur in the same capital recovery year ($m = q$). It is also assumed that the income tax change must occur before the CRF update ($\gamma \leq \mu$).

Because Case I and Case II are the same for capital recovery years 1 through $m-1$, the expression for F_{m-1} in (2.3) holds for this case and $K_{m-1}^{(e)} = K_0 F_{m-1}$. The income tax is updated in year m ($= q$) and the CRF is also updated in year m. Similar to the previous case the next step is to derive an expression for $K_q^{(e,1)}$, the equity investment remaining at the midpoint of service year q, after accounting for all revenue payments under the original CRF c_1. The revenue for year q at the original CRF c_1 is $\mu c_1 K_0$ and the income tax for this portion of the revenue is

$$\gamma(c_1K_0 - \delta_q K_0 - I_q)s_1 + (\mu - \gamma)(c_1K_0 - \delta_q K_0 - I_q)s_2$$

which can be restated as

$$\mu c_1 K_0 s_2 + \gamma(c_1 - \delta_q)K_0(s_1 - s_2) - \gamma I_q(s_1 - s_2) - \mu(\delta_q K_0 + I_q)s_2\,.$$

The revenue in year q prior to the CRF update net income taxes, a portion of the debt payment and return on equity is

$$\mu c_1 K_0(1-s_2) - \gamma\left(c_1-\delta_q\right)K_0(s_1-s_2) + \gamma I_q(s_1-s_2) + \mu\left(\delta_q K_0 s_2 + I_q s_2 - P\right) - K_{m-1}^{(e)} r_e$$

and the equity investment remaining at the midpoint of service year q, after accounting for all revenue payments under the original CRF c_1 is

(2.13)

$$\begin{aligned} K_q^{(e,1)} = K_{m-1}^{(e)}(1+r_e) - \mu c_1 K_0(1-s_2) + \gamma\left(c_1-\delta_q\right)K_0(s_1-s_2) - \gamma I_q(s_1-s_2) \\ - \mu\left(\delta_q K_0 s_2 + I_q s_2 - P\right). \end{aligned}$$

The next step is to find the CRF c_2 that satisfies the present value equation

$$K_q^{(e,1)} = \sum_{j=q}^{N} \frac{CF_j}{(1+r_e)^{j-q}}$$

where CF_j is the after cash flow to the equity investor in year j. From the point in time when the CRF is updated, Case I and Case II are identical so that equation (2.12) from Case I holds for Case II. Using (2.13) to replace $K_q^{(e,1)}$ in (2.12) results in

(2.14)

$$\begin{aligned} c_2 = \frac{r_e(1+r_e)^{N-q}}{(1-s_2)\left[(1+r_e)^{N-q+1} - 1 - \mu r_e(1+r_e)^{N-q}\right]} \Bigg\{ F_{m-1}(1+r_e) - \mu c_1(1-s_2) \\ + \gamma\left(c_1-\delta_q\right)(s_1-s_2) - \gamma\frac{I_q}{K_0}(s_1-s_2) - s_2\sum_{j=q}^{N}\frac{\delta_j}{(1+r_e)^{j-q}} \\ - s_2(1-E)\frac{r_d(1+r_d)^{q-3/2}}{(1+r_d)^N - 1}\left[(1+r_d)^{N-q+1}\left(\frac{(1+r_e)^{N-q+1}-1}{r_e(1+r_e)^{N-q}}\right)\right. \\ \left. - \frac{(1+r_e)^{N-q+1} - (1+r_d)^{N-q+1}}{(r_e-r_d)(1+r_e)^{N-q}}\right] + \frac{P}{K_0}\frac{(1+r_e)^{N-q+1}-1}{r_e(1+r_e)^{N-q}} \Bigg\} \end{aligned}$$

where F_{m-1} is as given in (2.3). Additional algebraic manipulation is necessary to show that the expression in (2.14) is equivalent to (2.1).

Equation (2.2) under the assumptions of this case $(1 < m = q)$ is

(2.15)

$$
\begin{aligned}
F_A = F_{m-1} &+ \gamma(c_1 - \delta_m)(s_1 - s_2)(1+\mathrm{r_e})^{-1} \\
&- (1-E)[\gamma s_1 + (1-\gamma)s_2]\frac{r_d(1+r_d)^{q-1}}{\sqrt{1+r_d}}\left(\frac{(1+r_d)^{N-q+1}-1}{(1+r_d)^N-1}\right)(1+r_e)^{-1} \\
&- (1-E)s_2\frac{r_d(1+r_d)^{q-1/2}}{(1+r_d)^N-1}\left[(1+r_d)^{N-q}\left(\frac{(1+r_e)^{-1}-1}{r_e}\right)\right. \\
&\left.- \frac{(1+r_e)^{-1}-(1+r_d)^{-1}}{r_e-r_d}\right].
\end{aligned}
$$

Simplifying the expression in (2.15) and noting by (3.5) that

$$I_q = (1-E)K_0\frac{r_d(1+r_d)^{q-1}}{\sqrt{1+r_d}}\left(\frac{(1+r_d)^{N-q+1}-1}{(1+r_d)^N-1}\right)$$

the expression in (2.15) can be restated as

(2.16)

$$F_A(1+r_e) = F_{m-1}(1+r_e) + \gamma\big(c_1-\delta_q\big)(s_1-s_2) - \gamma\frac{I_q}{K_0}(s_1-s_2)\,.$$

The expression for c_2 in (2.1) is obtained by replacing the shaded terms in (2.14) with $F_A(1+r_e)$ and replacing P using (3.5).

2.3.3 Case III: Tax rate change occurs in the first year, CRF is updated in a later year ($\mathbf{1 = m < q \leq N}$)

The income tax change occurs in the first year of capital recovery and the CRF is updated in a later year ($m < q$). The income tax payment in the year 1 is

$$\gamma(c_1K_0 - \delta_1K_0 - I_1)s_1 + (1-\gamma)(c_1K_0 - \delta_1K_0 - I_1)s_2$$

or equivalently

$$c_1K_0s_2 - \delta_1K_0s_2 + \gamma(c_1-\delta_1)K_0(s_1-s_2) - [\gamma s_1 + (1-\gamma)s_2]I_1.$$

The revenue net of taxes, the debt payment and return on equity for year 1 is

$$c_1K_0(1-s_2) + \delta_1K_0s_2 - \gamma(c_1-\delta_1)K_0(s_1-s_2) + [\gamma s_1 + (1-\gamma)s_2]I_1 - P - EK_0\big(\sqrt{1+r_e}-1\big)$$

where the last term of the expression represents a half year return on the initial equity investment (EK_0). The equity investment remaining at the midpoint of year 1, denoted to $K_1^{(e)}$, is equal to the initial equity investment EK_0 reduced by the year 1 revenue net of taxes, the debt payment and return on equity,

$$K_1^{(e)} = EK_0\sqrt{1+r_e} - \mathrm{c_1K_0(1-s_2)} - \delta_1 K_0 s_2 + \gamma(c_1-\delta_1)K_0(s_1-s_2) - [\gamma s_1 + (1-\gamma)s_2]I_1 + P\,.$$

The new tax rate s_2 is effective for year 2. The revenue net of taxes, the debt payment and return on equity for year 2, in the case that $q > 2$, is

$$c_1K_0(1-s_2) + \delta_2 K_0 s_2 + I_2 s_2 - P - K_1^{(e)} r_e$$

and the capital investment remaining at the midpoint of year 2 is

$$\begin{aligned}
K_2^{(e)} &= K_1^{(e)}(1+\mathrm{r_e}) - \mathrm{c_1K_0(1-s_2)} - \mathrm{\delta_2K_0s_2} - I_2s_2 + P \\
&= K_0E(1+r_e)^{3/2} - \mathrm{c_1}K_0(1-\mathrm{s_2})[(1+\mathrm{r_e})+1] - s_2K_0[\delta_1(1+\mathrm{r_e})+\delta_2] \\
&\qquad + \gamma(c_1-\delta_1)K_0(s_1-s_2)(1+\mathrm{r_e}) - [\gamma s_1+(1-\gamma)s_2]I_1(1+\mathrm{r_e}) - I_2s_2 \\
&\qquad + P[(1+\mathrm{r_e})+1]\,.
\end{aligned}$$

For year 3, the revenue net of taxes, the debt payment and return on equity, in the case that $q > 3$, is

$$c_1K_0(1-s_2) + \delta_3 K_0 s_2 + I_3 s_2 - P - K_2^{(e)} r_e$$

and the capital investment remaining at the midpoint of year $m + 2$ is

$$\begin{aligned}
K_3^{(e)} &= K_2^{(e)}(1+\mathrm{r_e}) - \mathrm{c_1K_0(1-s_2)} - \mathrm{\delta_3K_0s_2} - I_3s_2 + P \\
&= K_0E(1+r_e)^{5/2} - \mathrm{c_1}K_0(1-\mathrm{s_2})[(1+\mathrm{r_e})^2+(1+\mathrm{r_e})+1] \\
&\qquad - K_0s_2[\delta_1(1+\mathrm{r_e})^2+\delta_2(1+\mathrm{r_e})+\delta_3] + \gamma(c_1-\delta_1)K_0(s_1-s_2)(1+\mathrm{r_e})^2 \\
&\qquad - [\gamma s_1+(1-\gamma)s_2]I_1(1+\mathrm{r_e})^2 - s_2[I_2(1+\mathrm{r_e})+I_3] + P[(1+\mathrm{r_e})^2+(1+\mathrm{r_e})+1]\,.
\end{aligned}$$

Noting the pattern,

$$\begin{aligned}
K_{q-1}^{(e)} = K_0E(1+r_e)^{q-3/2} &- \mathrm{c_1}K_0(1-\mathrm{s_2})\sum_{j=0}^{q-2}(1+\mathrm{r_e})^j - K_0s_2\sum_{j=1}^{q-1}\delta_j(1+\mathrm{r_e})^{q-1-j} \\
&+ \gamma(c_1-\delta_1)K_0(s_1-s_2)(1+\mathrm{r_e})^{q-2} - [\gamma s_1+(1-\gamma)s_2]I_1(1+\mathrm{r_e})^{q-2} \\
&- s_2\sum_{j=2}^{q-1}I_j(1+\mathrm{r_e})^{q-1-j} + P\sum_{j=0}^{q-2}(1+\mathrm{r_e})^j\,.
\end{aligned}$$

Using equation (3.1) yields

$$\sum_{j=0}^{q-2}(1+\mathrm{r_e})^j = \frac{(1+\mathrm{r_e})^{q-1}-1}{\mathrm{r_e}}$$

and after rewriting

$$\sum_{j=2}^{q-1} I_j(1+\mathrm{r_e})^{q-1-j} = (1+\mathrm{r_e})^{q-1}\sum_{j=2}^{q-1} I_j\left(\frac{1}{1+\mathrm{r_e}}\right)^j$$

(3.6) yields

$$\sum_{j=2}^{q-1} I_j(1+\mathrm{r_e})^{q-1-j}$$
$$= (1-E)K_0\frac{r_d\sqrt{1+r_d}}{(1+r_d)^N-1}\left\{(1+r_d)^{N-1}\left(\frac{(1+\mathrm{r_e})^{q-2}-1}{r_e}\right)\right.$$
$$\left.-\left(\frac{(1+\mathrm{r_e})^{q-2}-(1+\mathrm{r_d})^{q-2}}{r_e-r_d}\right)\right\}.$$

Replacing the summations and using (3.5) to replace I_1 and P,

$$K_{q-1}^{(e)} = K_0\left\{E(1+r_e)^{q-3/2} - \mathrm{c_1}(1-\mathrm{s_2})\frac{(1+\mathrm{r_e})^{q-1}-1}{\mathrm{r_e}} - s_2\sum_{j=1}^{q-1}\delta_j(1+\mathrm{r_e})^{q-1-j}\right.$$
$$+\gamma(c_1-\delta_1)(s_1-s_2)(1+\mathrm{r_e})^{q-2}$$
$$-(1-E)[\gamma s_1+(1-\gamma)s_2]\left(\sqrt{1+r_d}-1\right)(1+\mathrm{r_e})^{q-2}$$
$$-(1-E)s_2\frac{r_d\sqrt{1+r_d}}{(1+r_d)^N-1}\left\{(1+r_d)^{N-1}\left(\frac{(1+\mathrm{r_e})^{q-2}-1}{r_e}\right)\right.$$
$$\left.\left.-\frac{(1+\mathrm{r_e})^{q-2}-(1+\mathrm{r_d})^{q-2}}{r_e-r_d}\right\}+(1-E)\left(\frac{r_d(1+r_d)^{N-1/2}}{(1+r_d)^N-1}\right)\frac{(1+\mathrm{r_e})^{q-1}-1}{\mathrm{r_e}}\right\}$$

or equivalently, $K_{q-1}^{(e)} = K_0F_A$ where F_A is obtained from (2.2) under the condition that $m=1$. This completes the proof that (2.1) holds for this case since the remainder of the proof would proceed exactly as in Case I beginning with equation (2.7).

2.3.4 Case IV: Tax rate change and CRF Update in first year ($1 = m = q \leq N$)

In Case IV the income tax change occurs in the first year of capital recovery and the CRF is also updated in the first year of capital recovery. The year 1 revenue from payments at the original CRF is equal to $\mu c_1 K_0$ and the associated income tax is

$$\gamma(c_1 K_0 - \delta_1 K_0 - I_1)s_1 + (\mu - \gamma)(c_1 K_0 - \delta_1 K_0 - I_1)s_2$$

which can be restated as

$$\mu c_1 K_0 s_2 + \gamma(c_1 - \delta_1)K_0(s_1 - s_2) - \gamma I_1(s_1 - s_2) - \mu(\delta_1 K_0 + I_1)s_2 \,.$$

The revenue in year 1 prior to the CRF update net income taxes, a portion of the debt payment and return on equity is

$$\mu c_1 K_0(1 - s_2) - \gamma(c_1 - \delta_1)K_0(s_1 - s_2) + \gamma I_1(s_1 - s_2) + \mu(\delta_1 K_0 s_2 + I_1 s_2 - P) - EK_0\left(\sqrt{1 + r_e} - 1\right)$$

where the last term of the expression is a half year return on the initial equity investment (EK_0). The equity investment remaining at the midpoint of service year 1, after accounting for all revenue payments under the original CRF c_1 is

(2.17)

$$K_1^{(e,1)} = K_0\left\{E\sqrt{1 + r_e} - \mu c_1(1 - s_2) + \gamma(c_1 - \delta_1)(s_1 - s_2) - \gamma\frac{I_1}{K_0}(s_1 - s_2) - \mu\left(\delta_1 s_2 + \frac{I_1}{K_0}s_2 - \frac{P}{K_0}\right)\right\}.$$

The updated CRF c_2 is determined by requiring the present value of the after tax cash flows to the equity investor, corresponding to revenue payments after the CRF update, is equal to $K_1^{(e,1)}$. The revenue after the CRF update in year 1 is $(1 - \mu)c_2 K_0$ and the after tax cash flow to the equity investors is

$$(1 - \mu)[c_2 K_0(1 - s_2) + \delta_1 K_0 s_2 + I_1 s_2 - P] \,.$$

The after tax cash flows for years $j = 2$ through N are

$$c_2 K_0(1 - s_2) + \delta_j K_0 s_2 + I_j s_2 - P \,.$$

The present value of the after tax cash flows, corresponding to revenue payments after the CRF update, is

$$(1-\mu)[c_2K_0(1-s_2)+\delta_1K_0s_2+I_1s_2-P]+\sum_{j=2}^{N}\frac{c_2K_0(1-s_2)+\delta_jK_0s_2+I_js_2-P}{(1+r_e)^{j-2}}$$

which can be restated as

(2.18)

$$c_2K_0(1-s_2)\sum_{j=1}^{N}\frac{1}{(1+r_e)^{j-1}}+K_0s_2\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^{j-1}}+s_2\sum_{j=1}^{N}\frac{I_j}{(1+r_e)^{j-1}}-P\sum_{j=1}^{N}\frac{1}{(1+r_e)^{j-1}}$$
$$-\mu[c_2K_0(1-s_2)+\delta_1K_0s_2+I_1s_2-P]\,.$$

Using (3.1)

$$\sum_{j=1}^{N}\frac{1}{(1+r_e)^{j-1}}=\frac{(1+r_e)^N-1}{r_e(1+r_e)^{N-1}}$$

and using (3.6)

$$\sum_{j=1}^{N}\frac{I_j}{(1+r_e)^{j-1}}=I_1$$
$$+(1-E)K_0\frac{r_d\sqrt{1+r_d}}{(1+r_d)^N-1}\left\{(1+r_d)^{N-1}\frac{(1+r_e)^{N-1}-1}{r_e(1+r_e)^{N-1}}\right.$$
$$\left.-\frac{(1+r_e)^{N-1}-(1+r_d)^{N-1}}{(r_e-r_d)(1+r_e)^{N-1}}\right\}.$$

Upon making the substitutions, the present value expression in (2.18) is restated as

(2.19)

$$\begin{aligned}
&c_2K_0(1-s_2)\frac{(1+r_e)^N-1}{r_e(1+r_e)^{N-1}}+K_0s_2\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^{j-1}}+s_2I_1\\
&\qquad+(1-E)s_2K_0\frac{r_d\sqrt{1+r_d}}{(1+r_d)^N-1}\left\{(1+r_d)^{N-1}\frac{(1+r_e)^{N-1}-1}{r_e(1+r_e)^{N-1}}\right.\\
&\qquad\left.-\frac{(1+r_e)^{N-1}-(1+r_d)^{N-1}}{(r_e-r_d)(1+r_e)^{N-1}}\right\}-P\frac{(1+r_e)^N-1}{r_e(1+r_e)^{N-1}}-\mu c_2K_0(1-s_2)\\
&\qquad-\mu[\delta_1K_0s_2+I_1s_2-P]\,.
\end{aligned}$$

[19]

Setting the expression in (2.19) equal to $K_1^{(e,1)}$ and solving for c_2 yields

(2.20)

$$\begin{aligned}
c_2=&\frac{r_e(1+r_e)^{N-1}}{(1-s_2)[(1+r_e)^N-1-\mu r_e(1+r_e)^{N-1}]}\left\{\frac{K_1^{(e,1)}}{K_0}-s_2\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^{j-1}}-s_2\frac{I_1}{K_0}\right.\\
&-(1-E)s_2\frac{r_d\sqrt{1+r_d}}{(1+r_d)^N-1}\left\{(1+r_d)^{N-1}\frac{(1+r_e)^{N-1}-1}{r_e(1+r_e)^{N-1}}\right.\\
&\left.\left.-\frac{(1+r_e)^{N-1}-(1+r_d)^{N-1}}{(r_e-r_d)(1+r_e)^{N-1}}\right\}+\frac{P}{K_0}\frac{(1+r_e)^N-1}{r_e(1+r_e)^{N-1}}+\mu\left[\delta_1s_2+\frac{I_1}{K_0}s_2-\frac{P}{K_0}\right]\right\}
\end{aligned}$$

Replacing $K_1^{(e,1)}$ in (2.20) with the expression in (2.17) and rearranging terms yields

[19] This equation holds for $N \geq 1$. If $N = 1$, the right hand side reduces to the portion of the after tax cash flow in year 1 after the CRF update.

(2.21)

$$c_2 = \frac{r_e(1+r_e)^{N-1}}{(1-s_2)[(1+r_e)^N - 1 - \mu r_e(1+r_e)^{N-1}]}\left\{E\sqrt{1+r_e} - \mu c_1(1-s_2) + \gamma(c_1-\delta_1)(s_1-s_2)\right.$$

$$- [\gamma s_1 + (1-\gamma)s_2]\frac{I_1}{K_0} - s_2\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^{j-1}}$$

$$- (1-E)s_2\frac{r_d\sqrt{1+r_d}}{(1+r_d)^N - 1}\left\{(1+r_d)^{N-1}\frac{(1+r_e)^{N-1}-1}{r_e(1+r_e)^{N-1}}\right.$$

$$\left.\left. - \frac{(1+r_e)^{N-1} - (1+r_d)^{N-1}}{(r_e-r_d)(1+r_e)^{N-1}}\right\} + \frac{P}{K_0}\frac{(1+r_e)^N - 1}{r_e(1+r_e)^{N-1}}\right\}.$$

To match the expression for c_2 in (2.1) additional algebraic manipulation is required. Using (3.7) and (3.8)

$$\frac{(1+r_e)^{N-1}-1}{r_e} = \frac{(1+r_e)^N - 1}{r_e} - (1+\mathrm{r_e})^{N-1}$$

and

$$\frac{(1+r_e)^{N-1} - (1+r_d)^{N-1}}{(r_e - r_d)} = \frac{(1+\mathrm{r_e})^N - (1+\mathrm{r}_d)^N}{(r_e-r_d)(1+r_d)} - \frac{(1+\mathrm{r_e})^{N-1}}{1+r_d}.$$

Upon making these substitutions in (2.21) and replacing I_1 and P using (3.5),

(2.22)

$$c_2 = \frac{r_e(1+r_e)^{N-1}}{(1-s_2)[(1+r_e)^N - 1 - \mu r_e(1+r_e)^{N-1}]}\left\{E\sqrt{1+r_e} - \mu c_1(1-s_2) + \gamma(c_1-\delta_1)(s_1-s_2)\right.$$

$$- (1-E)[\gamma s_1 + (1-\gamma)s_2]\left(\sqrt{1+r_d} - 1\right) - s_2\sum_{j=1}^{N}\frac{\delta_j}{(1+r_e)^{j-1}}$$

$$- (1-E)s_2\frac{r_d}{(1+r_d)^N - 1}\left(\frac{1}{\sqrt{1+r_d}}\right)\left\{(1+r_d)^N\left[\frac{(1+r_e)^N - 1}{r_e(1+r_e)^{N-1}}\right]\right.$$

$$\left. - \left[\frac{(1+\mathrm{r_e})^N - (1+\mathrm{r}_d)^N}{(r_e-r_d)(1+r_e)^{N-1}}\right]\right\} + (1-E)s_2\frac{r_d}{\sqrt{1+r_d}}$$

$$\left. + (1-E)\left(\frac{r_d(1+r_d)^{N-1/2}}{(1+r_d)^N - 1}\right)\frac{(1+r_e)^N - 1}{r_e(1+r_e)^{N-1}}\right\}.$$

Next note that using the expression for F_A in (2.2) with $1 = m = q$ shows that

$$F_A(1 + r_e) = E\sqrt{1 + r_e} + \gamma(c_1 - \delta_1)(s_1 - s_2) - (1 - E)[\gamma s_1 + (1 - \gamma)s_2]\left(\sqrt{1 + r_d} - 1\right) + (1 - E)s_2\frac{r_d}{\sqrt{1 + r_d}}.$$

The terms of $F_A(1 + r_e)$ are shaded in (2.22). Replacing the shaded terms in (2.22) with $F_A(1 + r_e)$ yields the expression for c_2 in (2.1).

3 Appendix - Useful Formulas

- A formula for the sum of a finite geometric series is given by

(3.1)

$$\sum_{j=H}^{W} v^j = \frac{v^H}{1-v}\left(1 - v^{W-H+1}\right).$$

H and W are positive integers and v is any number except one ($v \neq 1$). Equation (3.1) is validated by noting that if S is the sum on the left hand side of (3.1), then $S - vS = v^H - v^{W+1}$ and solving for S gives the right hand side of (3.1).

- Formulas for the debt payment and the interest portion of the debt payment, assuming uniform annual end of year payments, are given by

(3.2)

$$P = D\frac{r_d(1+r_d)^N}{(1+r_d)^N - 1}$$

$$I_j = Dr_d(1+r_d)^{j-1}\left(\frac{(1+r_d)^{N-j+1} - 1}{(1+r_d)^N - 1}\right), \quad j = 1, \cdots, N$$

where D is the amount of the loan, r_d is the interest rate and N is the term of the loan. The formula for P follows by noting that the present value of the debt payments must equal the debt principal,

$$D = \sum_{j=1}^{N} \frac{P}{(1+r_d)^j}.$$

The right hand side is a finite geometric series and using (3.1) can we restated as

$$D = P\frac{1}{r_d}\left[\frac{(1+r_d)^N - 1}{(1+r_d)^N}\right].$$

Solving for P yields the formula in (3.2).

The interest payment for the year j is the product of the debt interest rate and the outstanding debt principal at the end of year $j-1$. The outstanding principal at the end of year 1 is the original debt less the repayment of the principal at the end of year 1 or

$$D_1 = D_0 - (P - I_1)$$

where D_0 is the original debt (i.e. $D_0 = D$), P is the debt payment and I_1 is the interest portion of the debt payment. Noting that $I_1 = r_d D_0$, D_1 is restated as

(3.3)

$$D_1 = D_0(1 + r_d) - P\,.$$

The outstanding debt at the end of year 2 is

$$D_2 = D_1 - (P - I_2)$$

and $I_2 = r_d D_1$ so that

$$D_2 = D_1(1 + r_d) - P\,.$$

Then replacing D_1 using (3.3),

$$D_2 = D_0(1 + r_d)^2 - P[1 + (1 + r_d)]\,.$$

This pattern continues and the outstanding principal at the end of year $j - 1$ can be written as

$$D_{j-1} = D_0(1 + r_d)^{j-1} - P\sum_{k=1}^{j-1}(1 + r_d)^{k-1}\,.$$

Rewriting the sum as

$$\frac{1}{1 + r_d}\sum_{k=1}^{j-1}(1 + r_d)^k$$

and using (3.1) results in

$$D_{j-1} = D_0(1 + r_d)^{j-1} - P\,\frac{(1 + r_d)^{j-1} - 1}{r_d}\,.$$

Then replacing P and simplifying reveals

$$D_{j-1} = D_0(1 + r_d)^{j-1}\left(\frac{(1 + r_d)^{N-j+1} - 1}{(1 + r_d)^N - 1}\right).$$

The formula for I_j in (3.2) is the product of D_{j-1} and r_d.

- A formula for a sum of discounted or compounded interest payments, assuming uniform end of year payments, is given by

(3.4)

$$\sum_{j=H}^{W} I_j v^j = \frac{Dr_d}{(1+r_d)^N - 1}\Bigg\{(1+r_d)^N\left(\frac{v^H}{1-v}\right)(1 - v^{W-H+1}) \\ - \frac{1}{1+r_d}\left(\frac{[(1+r_d)v]^H}{1-(1+r_d)v}\right)(1 - [(1+r_d)v]^{W-H+1})\Bigg\}$$

where H and W are positive integers and v is any number except one ($v \neq 1$).[20] Equation (3.4) follows by replacing I_j using (3.2) and writing

$$\sum_{j=H}^{W} I_j v^j = \frac{Dr_d}{(1+r_d)^N - 1}\left\{(1+r_d)^N \sum_{j=H}^{W} v^j - \frac{1}{1+r_d}\sum_{j=H}^{W}[(1+r_d)v]^j\right\}$$

and then using (3.1) to rewrite the summations.

- Formulas for the debt payment and interest portion of the debt payment, assuming the half year convention and uniform annual payments, are given by

(3.5)

$$P = D\frac{r_d(1+r_d)^{N-1/2}}{(1+r_d)^N - 1}$$

$$I_1 = D\left(\sqrt{1+r_d} - 1\right)$$

$$I_j = D\frac{r_d(1+r_d)^{j-1}}{\sqrt{1+r_d}}\left(\frac{(1+r_d)^{N-j+1} - 1}{(1+r_d)^N - 1}\right), \quad j = 2, \cdots, N$$

where D is the amount of the loan, r_d is the interest rate and N is the term of the loan.[21] The year 1 interest payment reflects a half year of interest.

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20 The term v is typically a discount factor such as $v = 1/(1+r)$ or a compounding factor, $v = 1 + r$, but the formula holds for any value of v except one ($v \neq 1$).

21 The derivations are similar to the derivations for the end of year payment formulas in (3.2).

- A formula for a sum of discounted or compounded interest payments, for $H > 1$ and assuming the half year convention, is given by

(3.6)

$$\sum_{j=H}^{W} I_j v^j = \frac{Dr_d(1+r_d)^{-3/2}}{(1+r_d)^N - 1}\left\{(1+r_d)^{N+1}\left(\frac{v^H}{1-v}[1-v^{W-H+1}]\right)\right.$$
$$\left. - \frac{[(1+r_d)v]^H}{1-(1+r_d)v}(1-[(1+r_d)v]^{W-H+1})\right\}.$$

Equation (3.6) follows by replacing I_j using (3.5) and writing

$$\sum_{j=H}^{W} I_j v^j = \frac{Dr_d(1+r_d)^{-3/2}}{(1+r_d)^N - 1}\left\{(1+r_d)^{N+1}\sum_{j=H}^{W} v^j - \sum_{j=H}^{W}[(1+r_d)v]^j\right\}.$$

Using (3.1) to replace the summations, the right hand side of (3.6) is obtained.

- The following relationship holds:

(3.7)

$$\frac{(1+\mathrm{r_e})^W - 1}{r_e} = (1+\mathrm{r_e})^{W-1} + \frac{(1+\mathrm{r_e})^{W-1} - 1}{r_e}$$

where W is a positive integer, $r_e \neq -1$ and $r_e \neq 0$. This follows by noting that

$$\sum_{j=1}^{W}\left(\frac{1}{1+\mathrm{r_e}}\right)^j = \frac{1}{1+\mathrm{r_e}} + \sum_{j=2}^{W}\left(\frac{1}{1+\mathrm{r_e}}\right)^j$$

and then using (3.1) to replace the summations.

- The following relationship holds:

(3.8)

$$\frac{(1+\mathrm{r_e})^W - (1+\mathrm{r}_d)^W}{r_e - r_d} = (1+\mathrm{r_e})^{W-1} + (1+r_d)\frac{(1+\mathrm{r_e})^{W-1} - (1+\mathrm{r_d})^{W-1}}{r_e - r_d}.$$

where W is a positive integer, $r_e \neq -1$ and $r_e \neq r_d$. This follows by noting that

$$\sum_{j=1}^{W}\left(\frac{1+r_d}{1+\mathrm{r_e}}\right)^j = \frac{1+r_d}{1+\mathrm{r_e}} + \sum_{j=2}^{W}\left(\frac{1+r_d}{1+\mathrm{r_e}}\right)^j.$$

and then using (3.1) to replace the summations.

Attachment V

Capital Recovery Factors (CRF)
Weighted Average Cost of Capital Approach
Technical Reference

Monitoring Analytics, LLC

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Table of Contents

1 The Basics of CRF

A capital recovery factor (CRF) is used to convert the principal of a capital investment (K) into an equivalent stream of uniform payments. A typical CRF formula found in engineering economics textbooks is given in equation (1.1).[1]

(1.1)

$$CRF = \frac{r(1+r)^N}{(1+r)^N - 1}$$

Variable r is the rate of return and N is the term (in years) over which the investment will be recovered. At the end of each year during the recovery term, the investor receives a payment equal to the product of the CRF and the investment. For example, consider a $1 million dollar investment. Assume the rate of return is 12 percent and the capital recovery term is five years. The CRF value given by (1.1) is 0.277410 and the uniform annual payment, to be paid at the end of each year of the five year recovery period, is $277,410. Table 1-1 shows the corresponding cash flow. The first payment is received at the end of the first capital recovery year. The payment covers the return on investment equal to $120,000, leaving $157,410 for repayment of the investment principal. The return on investment at the end of the first year is the product of the investment amount ($1 million) and the rate of return (12 percent). The return on investment at the end of year 2 is the product of the rate of return and the outstanding principal at the end of year 1. At the end of year 2, the difference between the annual revenue and the return on investment is $176,299 which goes toward repayment of the investment principal. The cash flow components for years 3 through 5 are analogous to the year 2 cash flow. At the end of year 5, the investment principal has been returned. The cash flow summary in Table 1-1 shows that the annual revenue determined by the CRF in (1.1) has provided for the return on and return of the $1 million investment.

1 For example, see pages 21-22 in "Economic Evaluation and Investment Decision Methods," Stermole, F.J. and Stermole, J.M. (1993).

Table 1-1 Investment cash flow for the basic CRF

Capital Recovery Year	1	2	3	4	5
Revenue	$277,410	$277,410	$277,410	$277,410	$277,410
Return on investment	$120,000	$101,111	$79,955	$56,260	$29,722
Repayment of investment principal	$157,410	$176,299	$197,455	$221,149	$247,687
Outstanding investment principal	$842,590	$666,291	$468,837	$247,687	$0

To derive (1.1) the CRF is denoted by c and the annual payment is defined as cK, the product of the CRF and the investment capital. The value of c is determined by requiring the present value of the annual payments is equal to the investment capital. The following equation captures this requirement,

$$K = \sum_{j=1}^{N} \frac{cK}{(1+r)^j}$$

which can be restated as

$$K = cK \sum_{j=1}^{N} \left(\frac{1}{1+r}\right)^j .$$

The summation in the equation is a finite geometric series. A formula for the sum of a finite geometric series is given by (3.1) in the appendix. Using (3.1) with $H = 1$, $W = N$ and $v = 1/(1+r)$ yields

$$\sum_{j=1}^{N} \left(\frac{1}{1+r}\right)^j = \frac{(1+r)^N - 1}{r(1+r)^N} .$$

Upon replacing the summation in the present value equation

$$K = cK \left(\frac{(1+r)^N - 1}{r(1+r)^N}\right)$$

and solving for c produces (1.1).

1.1 After tax CRF

The CRF value in (1.1) is a before tax calculation since it does not account for income taxes. The revenue that results from a capital investment is taxable income. The payment obtained by multiplying the capital investment amount K by the CRF in equation (1.1) would be too low in cases where the revenue is taxable. The goal, in the presence of taxes, is to have a CRF for which the product of the CRF and the investment K yields an annual payment that will provide the necessary and sufficient level of revenue to cover the investors' annual income tax payments, and the return on and return of the capital investment. The CRF and the associated annual revenue payment, can be determined by solving an equation where the present value of the after tax cash flows resulting from the annual revenue payments is equal to the initial capital investment.

The composition of the after tax cash flow is dependent upon the capital budgeting model. The weighted average cost of capital (WACC) model is used in the sections that follow.[2] The before tax WACC rate is a weighted average of the rate of return on equity and the interest rate on debt, where the weights reflect the debt to equity ratio.

$$\text{Before tax WACC } = \frac{E}{K} r_e + \frac{D}{K} r_d$$

The equity funding percent is represented by E, D is the debt funding percent ($E + D = 1$), r_e is the rate of return on equity and r_d is the interest rate on debt. The after tax WACC rate incorporates the income tax shield associated with debt investments,

$$\text{After tax WACC} = \frac{E}{K} r_e + \frac{D}{K} (1 - s) r_d$$

where s is the effective income tax rate. The WACC approach to capital budgeting discounts the after tax cash flow at the after tax WACC rate. The annual revenue payment is defined to the product of the CRF (c) and the capital investment (K). The income tax for year j includes an offset for deprecation and is equal to $(cK - \delta_j K)s$ where δ_j is the depreciation factor for year j. The after tax cash flow for year j is $cK - (cK - \delta_j K)s$ or equivalently

$$CF_j = cK(1 - s) + \delta_j K s \, .$$

The CRF is the value that satisfies the following equation where the investment capital is equal to present value of the after tax cash flow.

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2 Additional details on the WACC approach to capital budgeting can be found in Section 17.3 in "Corporate Finance," Ross, Westerfield, Jaffe, 4th Edition, 1996.

$$K = \sum_{j=1}^{N} \frac{CF_j}{(1+r)^j}$$

Parameter r is after tax WACC rate. This formulation assumes the revenue and income tax payments occur at the end of the year. The model parameters are defined in Table 1-2. Replacing CF_j leads to the following restatement of the present value equation

$$K = cK(1-s)\sum_{j=1}^{N} \frac{1}{(1+r)^j} + Ks\sum_{j=1}^{N} \frac{\delta_j}{(1+r)^j}$$

Using (3.1)

$$\sum_{j=1}^{N} \frac{1}{(1+r)^j} = \frac{(1+r)^N - 1}{r(1+r)^N}$$

and substituting into the previous equation results in

$$K = cK(1-s)\frac{(1+r)^N - 1}{r(1+r)^N} + Ks\sum_{j=1}^{N} \frac{\delta_j}{(1+r)^j}$$

.

Solving for c yields

(1.2)

$$c = \frac{r(1+r)^N}{(1-s)[(1+r)^N - 1]}\left\{1 - s\sum_{j=1}^{N} \frac{\delta_j}{(1+r_e)^j}\right\}.$$

Table 1-2 Parameter descriptions for the WACC capital budgeting model

Parameter	Description
r	After tax weighted average cost of capital (AT WACC)
s	Effective income tax rate
N	Capital investment recovery term
δ_j	Depreciation factor for year j

Substituting the parameter values shown in Table 1-3 into the CRF formula, assuming a five year capital recovery period and straight line depreciation yields a CRF of 0.287071. With a capital investment of $1 million, the annual payment is $287,071.

Table 1-3 Financial parameter and tax assumptions[3]

Parameter	Parameter Value
Equity funding percent	50.000%
Debt funding percent	50.000%
Rate of return on equity	12.000%
Debt interest rate	7.000%
Federal income tax rate	36.000%
State income tax rate	9.000%
Effective income tax rate	41.760%
After tax WACC	8.038%

Table 1-4 provides a cash flow summary for a $1 million capital investment with a five year capital recovery period that uses straight line depreciation. The revenue for each year, equal to the product of the CRF and the capital investment amount is $287,071. The gross tax payment for each year is equal to the effective income tax rate times the revenue net depreciation. The return on investment is equal the product of the outstanding investment principal and the after tax WACC rate.[4] Under the WACC approach, the income tax shield that accompanies a debt investment is reflected in the return on investment.[5] After accounting for the income tax payment, and the return on investment in year 1, $170,326 is available for repayment of the investment principal. Under the WACC approach, repayment of the investment is split according to the debt to equity ratio. In this example, 50 percent or $85,163 goes to repayment of debt and the same

[3] The effective tax rate (parameter s in the formula) is equal to *State Tax Rate + Federal Tax Rate x (1-State Tax Rate).*

[4] The outstanding or remaining investment refers to the investment principal that has not been repaid.

[5] Return on investment is equal to the sum of the return on equity and interest on debt, less the income tax shield. The income tax shield is equal to the product of the effective income tax rate and the interest on debt.

goes to the equity investors. The outstanding debt principal at the end of the first year is $414,837 and the outstanding or remaining equity investment is the same due to the assumed 50-50 debt to equity ratio. The year 2 return on investment is the product of the after tax WACC rate and the remaining investment (both equity and debt) at the end of year 1. The revenue in excess of income taxes and return on investment is $184,017 at the end of year 2 and this amount is split evenly toward repayment of the debt and equity investments. The cash flows for years 3 through 5 are analogous to the year 2 cash flow.

Table 1-4 Cash flow summary for 5 year, $1 million investment with straight line depreciation[6]

Capital Recovery Year	1	2	3	4	5
Revenue	$287,071	$287,071	$287,071	$287,071	$287,071
Depreciation	$200,000	$200,000	$200,000	$200,000	$200,000
Gross income tax	$36,361	$36,361	$36,361	$36,361	$36,361
Return on Investment	$80,384	$66,693	$51,900	$35,919	$18,654
Revenue in excess of taxes and return on investment	$170,326	$184,017	$198,810	$214,791	$232,056
Repayment of debt principal	$85,163	$92,009	$99,405	$107,395	$116,028
Repayment of equity investment	$85,163	$92,009	$99,405	$107,395	$116,028
Debt Remaining	$414,837	$322,828	$223,424	$116,028	$0
Equity Remaining	$414,837	$322,828	$223,424	$116,028	$0

After the final revenue payment in year 5, the remaining equity investment and the outstanding debt are reduced to $0. Summing horizontally across the debt remaining row and the equity remaining row produces $500,000 for each, reflecting the 1:1 debt to equity ratio in Table 1-3. This example illustrates that the revenue payment determined by the CRF provides the necessary and sufficient annual revenue to pay the income taxes associated with the revenue payment as well as the required return on and return of the capital investment.

1.2 Half Year Convention

The revenue and tax payments would likely be made on a monthly or quarterly basis rather than occurring at the end of the year. A better model with respect to the timing of the revenue and tax payments is obtained by assuming the revenue and tax payments occur at the midpoint of each year. To derive a CRF corresponding to midyear revenue and tax payments, the present value

6 WACC model with end of year revenue and tax payments.

equation from the previous section is modified to reflect the new timing assumption. Each after tax cash flow amount is assumed to occur a half year earlier than in the previous model. The revised present value equation is

$$K = \sum_{j=1}^{N} \frac{CF_j}{(1+r)^{j-0.5}} \,,$$

or equivalently,

$$\mathrm{K} = \sqrt{1+r} \sum_{\mathrm{j}=1}^{\mathrm{N}} \frac{CF_j}{(1+r)^{\mathrm{j}}} \,.$$

Making the substitution, $CF_j = cK(1-s) + \delta_j Ks$,

$$\mathrm{K} = cK(1-s)\sqrt{1+r} \sum_{\mathrm{j}=1}^{\mathrm{N}} \left(\frac{1}{1+r}\right)^{j} + Ks\sqrt{1+r} \sum_{\mathrm{j}=1}^{\mathrm{N}} \frac{\delta_j}{(1+r)^{\mathrm{j}}} \,.$$

Using (3.1)

$$\sum_{\mathrm{j}=1}^{\mathrm{N}} \left(\frac{1}{1+r}\right)^{j} = \frac{(1+r)^N - 1}{r(1+r)^N} \,.$$

Making the replacement and solving for c yields equation (1.3).

(1.3)

$$c = \frac{r(1+r)^N}{(1-s)[(1+r)^N - 1]} \left\{ \frac{1}{\sqrt{1+r}} - s \sum_{\mathrm{j}=1}^{\mathrm{N}} \frac{\delta_j}{(1+r)^{\mathrm{j}}} \right\} .$$

The formula in (1.3) returns the after tax CRF assuming the half year convention and the WACC approach. Using the parameter values in Table 1-3, with a five year capital cost recovery period and straight line depreciation, (1.3) **Error! Reference source not found.**yields a CRF of 0.270747. If the initial capital investment $1 million, the annual payment is $270,747. Table 1-5 shows the corresponding cash flow summary.

Table 1-5 Cash flow summary for 5 year, $1 million investment with half year convention and straight line depreciation

Capital Recovery Year	1	2	3	4	5
Revenue	$270,747	$270,747	$270,747	$270,747	$270,747
Depreciation	$200,000	$200,000	$200,000	$200,000	$200,000
Gross Income Tax	$29,544	$29,544	$29,544	$29,544	$29,544
Return on investment	$39,415	$64,163	$49,932	$34,557	$17,946
Revenue in excess of taxes and return on investment	$201,788	$177,039	$191,271	$206,646	$223,257
Repayment of debt principal	$100,894	$88,520	$95,635	$103,323	$111,628
Repayment of equity investment	$100,894	$88,520	$95,635	$103,323	$111,628
Debt Remaining	$399,106	$310,586	$214,951	$111,628	$0
Equity Remaining	$399,106	$310,586	$214,951	$111,628	$0

The calculation of the values in Table 1-5 is identical to the corresponding values in Table 1-4 except that the year 1 return on investment reflects a half year period. The return on investment in year 1 is equal to the product of the investment and the half year rate of return $\sqrt{1+\mathrm{r}}-1$. The cash flow summary shows that the revenue payment determined by the CRF in (1.3) provides the necessary and sufficient level of revenue to pay the income taxes associated with the revenue payment as well as the required return on and return of the capital investment.

Changing the depreciation assumption to 3 year MACRS produces a CRF of 0.259347. The MACRS depreciation factors are shown in Table 1-9. The lower CRF relative to the straight line depreciation example reflects the lower tax payment under MACRS due to the accelerated depreciation schedule. In years 1 and 2, the tax payment in Table 1-6 is negative due to the accelerated depreciation assumption.[7] The cash flow summary in Table 1-6 shows that the revenue payment determined by the CRF, using 3 year MACRS depreciation, is at the necessary and sufficient level to provide for the taxes associated with the revenue payment as well as the required return on and return of the capital investment.

[7] It is assumed that the investor would use the negative tax liability from this project as an offset against the tax liability resulting from other revenue.

Table 1-6 Cash flow summary for 5 year, $1 million investment with 3 year MACRS

Capital Recovery Year	1	2	3	4	5
Revenue	$259,347	$259,347	$259,347	$259,347	$259,347
Depreciation	$333,300	$444,500	$148,100	$74,100	$0
Gross Income Tax	($30,883)	($77,320)	$46,457	$77,359	$108,303
Return on investment	$39,415	$60,223	$38,001	$23,942	$11,238
Revenue in excess of taxes and return on investment	$250,815	$276,445	$174,890	$158,046	$139,806
Repayment of debt principal	$125,407	$138,222	$87,445	$79,023	$69,903
Repayment of equity investment	$125,407	$138,222	$87,445	$79,023	$69,903
Debt Remaining	$374,593	$236,370	$148,926	$69,903	$0
Equity Remaining	$374,593	$236,370	$148,926	$69,903	$0

Assuming 15 year MACRS depreciation results in a CRF of 0.360403. The corresponding cash flow summary is given in Table 1-7. The CRF is higher in this case, relative to the straight line and 3 year MACRS examples, because the income tax payment is higher. The depreciation offset is lower in each year and the investment is not fully depreciated during the capital recovery term.

Table 1-7 Cash flow summary for 5 year, $1 million investment with 15 year MACRS

Capital Recovery Year	1	2	3	4	5
Revenue	$360,403	$360,403	$360,403	$360,403	$360,403
Depreciation	$50,000	$95,000	$85,500	$76,950	$69,255
Gross Income Tax	$129,624	$110,832	$114,799	$118,370	$121,583
Return on investment	$39,415	$65,001	$50,165	$34,455	$17,769
Revenue in excess of taxes and return on investment	$191,363	$184,569	$195,438	$207,578	$221,051
Repayment of debt principal	$95,682	$92,285	$97,719	$103,789	$110,525
Repayment of equity investment	$95,682	$92,285	$97,719	$103,789	$110,525
Debt Remaining	$404,318	$312,034	$214,314	$110,525	$0
Equity Remaining	$404,318	$312,034	$214,314	$110,525	$0

Assuming 100 percent bonus depreciation results in a CRF of 0.247761. The corresponding cash flow summary is given in Table 1-8.

Table 1-8 Cash flow summary for 5 year, $1 million investment with bonus depreciation

Capital Recovery Year	1	2	3	4	5
Revenue	$247,761	$247,761	$247,761	$247,761	$247,761
Depreciation	$1,000,000	$0	$0	$0	$0
Gross Income Tax	($314,135)	$103,465	$103,465	$103,465	$103,465
Return on investment	$39,415	$38,385	$29,871	$20,673	$10,736
Revenue in excess of taxes and return on investment	$522,481	$105,911	$114,425	$123,623	$133,560
Repayment of debt principal	$261,240	$52,956	$57,212	$61,811	$66,780
Repayment of equity investment	$261,240	$52,956	$57,212	$61,811	$66,780
Debt Remaining	$238,760	$185,804	$128,591	$66,780	$0
Equity Remaining	$238,760	$185,804	$128,591	$66,780	$0

In each example, the annual revenue, equal to the product of the capital investment and the CRF is the necessary and sufficient revenue amount to cover the income tax liability and the return on and return of the investment capital.

Table 1-9 Modified Accelerated Cost Recovery System (MACRS) with half year convention[8]

Year	3 year Depreciation Factors	5 year Depreciation Factors	10 year Depreciation Factors	15 year Depreciation Factors	20 year Depreciation Factors
1	33.33%	20.00%	10.00%	5.00%	3.750%
2	44.45%	32.00%	18.00%	9.50%	7.219%
3	14.81%	19.20%	14.40%	8.55%	6.677%
4	7.41%	11.52%	11.52%	7.70%	6.177%
5		11.52%	9.22%	6.93%	5.713%
6		5.76%	7.37%	6.23%	5.285%
7			6.55%	5.90%	4.888%
8			6.55%	5.90%	4.522%
9			6.56%	5.91%	4.462%
10			6.55%	5.90%	4.461%
11			3.28%	5.91%	4.462%
12				5.90%	4.461%
13				5.91%	4.462%
14				5.90%	4.461%
15				5.91%	4.462%
16				2.95%	4.461%
17					4.462%
18					4.461%
19					4.462%
20					4.461%
21					2.231%

8 See Appendix A, Table A-1, IRS Publication 946, United States Department of Treasury (2020).

2 Updating the CRF to reflect a change in the tax law

A change in the income tax rate or the eligible depreciation offset can have a significant impact on the recovery of a capital investment. The Tax Cuts and Jobs Act (TCJA) of 2017 lowered the federal corporate income tax rate from 35 percent to 21 percent.[9] [10] The TCJA also introduced bonus depreciation that allows for increased depreciation during the first year of an asset's service life.[11]

When the tax law changes during the capital cost recovery period or the initial CRF is incorrect, it is necessary to update the CRF to avoid an incorrect capital cost recovery.[12] The updated CRF should adhere to the previously stated requirements that revenue payments resulting from the CRF provide for the return on and return of the capital investment in addition to covering the income tax liabilities associated with the revenue payments. The updated CRF will need to incorporate any lag between the change in the income tax rate and the change in the CRF. The procedure for establishing the updated CRF can be described as a two step process (1) the outstanding investment principal is determined as of the effective date of the updated CRF and (2) an updated CRF is calculated based on the remaining capital recovery term and the outstanding investment principal.

2.1 Updated CRF for the weighted average cost of capital (WACC) model

The updated model incorporates a change in the tax rate and a change in the CRF. The two changes are treated independently in order to allow for the delayed implementation of a revised

9 Tax Cuts and Jobs Act, Pub. L. No. 115-97, 131 Stat. 2096, Stat. 2105 (2017).

10 26 U.S. Code §11(b).

11 Bonus depreciation is 100 percent for capital investments placed in service after September 27, 2017 and before January 1, 2023. Bonus depreciation is 80 percent for capital investments placed in service after December 31, 2022 and before January 1, 2024, and the bonus depreciation level is reduced by 20 percent for each subsequent year through 2026. Capital investments placed in service after December 31, 2026 are not eligible for bonus depreciation. See 26 U.S. Code §168(k)(6)(A).

12 PJM black start resources that began capital cost recovery between January 1, 2018 and June 6, 2021 were assigned an incorrect CRF based on outdated and incorrect income tax rules.

CRF. Variable m represents the capital recovery year during which the tax change occurs, and variable q represents the first capital recovery year during which the updated CRF is effective.[13] Variable γ ($0 \leq \gamma < 1$) is the fractional portion of year m for which the old tax rate is applicable. Variable μ ($0 \leq \mu < 1$) is the fractional portion of year q for which the old CRF is applicable. For example, consider a black start unit that began service on December 1, 2017 and assume the updated CRF will be effective on November 1, 2021. The TCJA income tax rate change was effective on January 1, 2018 which was one month into the capital recovery term, so that m is 1 and γ is 0.0833. The updated CRF became effective eleven months into year 4 of the capital recovery term, so that q is 4 and μ is 0.9167. Table 2-1 provides a description of the model parameters. The initial effective income tax rate is s_1 and the initial CRF is c_1. In year m the effective income tax rate changes to s_2, and in year q the CRF is updated to c_2. The after tax WACC rate is dependent upon the income tax rate. Parameter r_1 represents the after tax WACC rate corresponding to the initial income tax rate s_1 and r_2 is the after tax WACC rate corresponding to the updated income tax rate s_2.

Table 2-1 Parameter descriptions for the updated CRF for the WACC model

Parameter	Description
E	Equity funding percent
r_1	After tax WACC corresponding to the initial effective income tax rate
r_2	After tax WACC corresponding to the updated effective income tax rate
s_1	Initial effective income tax rate
s_2	Updated effective income tax rate
c_1	Initial CRF
c_2	Updated CRF
N	Capital investment recovery period
m	Recovery year during which the income tax rate is updated
γ	Portion of recovery year m for which tax rate s_1 applies
q	Recovery year during which the CRF is updated
μ	Portion of recovery year q for which CRF c_1 applies
δ_j	Depreciation factor for year j

13 The income tax change occurs prior to or coincident with the CRF update ($m \leq q$).

The updated CRF is given by equation (2.1).

(2.1)

$$c_2 = \left\{ \frac{r_2(1+r_2)^{N-q}}{(1-s_2)\left[(1+r_2)^{N-q+1} - 1 - \left(\frac{1+r_1}{1+r_2}\right)^{G} \mu r_2 (1+r_2)^{N-q} + \left[\left(\frac{1+r_1}{1+r_2}\right)^{G} - 1\right] r_2(1+r_2)^{N-q}\right]} \right\}$$

$$\cdot \left\{ \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-0.5} (1+r_1)^{m-.5}(1+r_2)^{q-m} \right.$$

$$- c_1(1-s_1)\left(\frac{(1-r_1)^{m-1}-1}{r_1}\right)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-0.5}(1+r_1)(1+r_2)^{q-m}$$

$$- s_1 X \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-0.5}(1+r_1)(1+r_2)^{q-m} - c_1(1-s_2)\left(\frac{(1+r_2)^{q-m}-1}{r_2}\right)(1+r_2)$$

$$- s_2 Y(1+r_2) + \gamma(c_1-\delta_m)(s_1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{H}(1+r_2)^{q-m}$$

$$- c_1(1-s_2)\left[\left(\frac{1+r_1}{1+r_2}\right)^{F} - 1\right](1+r_2)^{q-m} - \delta_m s_2 \left[\left(\frac{1+r_1}{1+r_2}\right)^{H} - 1\right](1+r_2)^{q-m}$$

$$\left. - \mu c_1(1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{G} - s_2 \sum_{j=q}^{N} \frac{\delta_j}{(1+r_2)^{j-q}} \right\}$$

where

$$F = \begin{cases} 0 & \gamma \le 0.5 \\ \gamma - 0.5 & \gamma > 0.5\,, m < q \\ 0 & \gamma > 0.5\,, m = q \end{cases}$$

$$G = \begin{cases} 0 & \gamma \le 0.5 \\ 0 & \gamma > 0.5\,, m < q \\ \gamma - 0.5 & \gamma > 0.5\,, m = q \end{cases}$$

$$H = \begin{cases} 0 & \gamma \le 0.5 \\ \gamma - 0.5 & \gamma > 0.5 \end{cases}$$

$$X = \begin{cases} 0, & m = 1 \\ \sum_{j=1}^{m-1} \delta_j (1+r_1)^{m-1-j}, & m > 1 \end{cases}$$

$$Y = \begin{cases} 0, & m = q \\ \sum_{j=m}^{q-1} \delta_j (1+r_2)^{q-1-j}, & m < q \end{cases} .$$

As an example, consider a $1 million capital investment with a five year capital cost recovery period that began on May 1, 2016 and assume the updated CRF has an effective date of January 1, 2019. The cost recovery period began prior to the TCJA. Assuming the initial CRF was calculated using the parameter values in Table 1-3 and 15 year MACRS depreciation, equation (1.3) returns a CRF of 0.360403. The corresponding annual revenue payment is $360,403. The tax rate change has an effective date of January 1, 2018 which is eight months into the second capital cost recovery year ($m = 2, \gamma = 0.6667$). Table 2-2 shows the result if the CRF is not updated to reflect the change in the federal income tax rate.

Table 2-2 Cash flow summary with no update to the CRF under FTE, $1 million investment

Capital Recovery Year	1	2	3	4	5
Revenue	$360,403	$360,403	$360,403	$360,403	$360,403
Depreciation	$50,000	$95,000	$85,500	$76,950	$69,255
Gross Income Tax	$129,624	$98,756	$77,275	$79,679	$81,842
Return on investment	$39,415	$65,001	$51,630	$32,403	$11,256
Revenue in excess of taxes and return on investment	$191,363	$196,645	$231,498	$248,321	$267,305
Repayment of debt principal	$95,682	$98,323	$115,749	$124,160	$66,086
Repayment to equity investment	$95,682	$98,323	$115,749	$124,160	$66,086
Debt Remaining	$404,318	$305,996	$190,247	$66,086	$0
Equity Remaining	$404,318	$305,996	$190,247	$66,086	$0
Excess Revenue to equity investors	$0	$0	$0	$0	$135,133

The revenue remains at the annual rate of $360,403 for the duration of the capital recovery period. The first year cash flow is identical to Table 1-7, which was based on the same initial assumptions. For years 2 through 5 , the cash flow in Table 2-2 reflects the lower federal income tax rate. The repayment of the investment in years 2 through 4 is higher in comparison to Table 1-7 and the remaining equity is reduced to $132,173 (split evenly between debt and equity) in year 4. In year 5, the remaining debt and equity are repaid and the equity investors receive an additional $135,133. The equity investor's rate of return, intended to be 12.0 percent, is 19.1 percent. Table 2-2 shows that the failure to update the CRF will lead to over recovery for the equity investor.

Table 2-3 shows the cash flow that results when the CRF is updated using (2.1). The effective date of the updated CRF is January 1, 2019 ($q = 3, \mu = 0.6667$). Using (2.1) the updated CRF value is 0.284537 and the updated annual revenue amount is $284,537. In recovery year 3, the annual revenue rate is reduced to $284,537.[14] Using the updated CRF has eliminated the excess recovery and the equity investor's rate of return is 12.0 percent. Table 2-3 shows that the CRF determined by (2.1) provides the necessary and sufficient revenue to pay the income tax liability and the return on and the return of capital investment.

Table 2-3 Cash flow summary with an updated CRF under FTE, $1 million investment

Capital Recovery Year	1	2	3	4	5
Revenue	$360,403	$360,403	$335,114	$284,537	$284,537
Depreciation	$50,000	$95,000	$85,500	$76,950	$69,255
Gross Income Tax	$129,624	$98,756	$70,167	$58,353	$60,516
Return on investment	$39,415	$65,001	$51,630	$33,952	$17,581
Revenue in excess of taxes and return on investment	$191,363	$196,645	$213,318	$192,233	$206,441
Repayment of debt principal	$95,682	$98,323	$106,659	$96,116	$103,220
Repayment to equity investment	$95,682	$98,323	$106,659	$96,116	$103,220
Debt Remaining	$404,318	$305,996	$199,337	$103,220	$0
Equity Remaining	$404,318	$305,996	$199,337	$103,220	$0
Excess Revenue to equity investors	$0	$0	$0	$0	$0

2.2 Generalized Formula

The CRF formula given in (2.1) is a generalization or extension of the CRF formula in (1.3). The extended formula in (2.1) reduces to the formula in (1.3) under the appropriate parameter settings. Formula (1.3) returns a CRF value for a new investment that is just beginning the capital recovery period. To get an initial CRF value from (2.1) the income tax rate change is assumed to have occurred on or before the investment was placed into service and the CRF update is assumed to have occurred at the beginning of the capital recovery term. The corresponding settings for the parameters are $m = 1$, $q = 1$, $\gamma = 0$, and $\mu = 0$. Under these assumptions, parameters F, G, H, X and Y are all zero and (2.1) reduces to

$$c_2 = \left\{\frac{r_2(1+r_2)^{N-1}}{(1-s_2)[(1+r_2)^N - 1]}\right\}\left\{\sqrt{1+r_2} - s_2 \sum_{j=q}^{N} \frac{\delta_j}{(1+r_2)^{j-1}}\right\}.$$

14 The revenue for year 3 reflects eight months at the original CRF and four months at the updated CRF.

Further algebraic manipulation, multiplying the first bracketed term by $(1 + r_2)$ and the second bracketed term by $1/(1 + r_2)$, produces (1.3).

Taking this analysis one step further, if the income tax rate is zero ($s_2 = 0$), the formula further reduces to

$$c_2 = \left(\frac{1}{\sqrt{1 + r_2}}\right)\frac{r_2(1 + r_2)^N}{(1 + r_2)^N - 1}.$$

This is the formula for a before tax CRF, assuming the half year convention, for an N year capital recovery term with rate of return r_2.[15]

2.3 Derivation of the updated CRF under the WACC model

The formula for the updated CRF in (2.1) is derived in a series of cases where assumptions are made regarding the timing of the change in the income tax rules and when the CRF is updated to reflect this change. The derivations for all cases will follow the same process. The outstanding investment at the midpoint of the capital recovery year during which the CRF is updated is determined. This value is denoted by $K_q^{(1)}$. The updated CRF value c_2, is determined by requiring that the present value of the after tax cash flow resulting from revenue payments under the updated CRF c_2 is equal to $K_q^{(1)}$. The first case assumes the income tax rule change occurred sometime after the first capital recovery year ($m > 1$) and the CRF is updated in a future year ($m < q$). Working through the cases, the timing assumptions are relaxed and it is shown that (2.1) holds for $1 \leq m \leq q \leq N$. Because the after tax WACC rate changes when the income tax rate changes, each case will be split into two subcases based on whether or not the income tax change occurs at the midpoint or prior to the midpoint of year m ($\gamma \leq 0.5$) or during the latter half of the year m ($\gamma > 0.5$).

15 Compare to (1.1) which assumes end of year payments.

2.3.1 Case I: Tax rate change occurs in year 2 or later and the CRF is updated in the following year or later ($1 < m < q \leq N$)

In the derivation that follows, the capital investment remaining at the midpoint of year $m-1$ is first established. The income tax rate change occurs in the following year and the derivation then splits into two subcases based on the whether the change occurs on or before the midpoint of year m or during the latter half of year m. The return on investment calculation for the two subcases differs because the after tax WACC rate changes when the income tax rate changes. Derivations of each subcase proceed by establishing the capital investment remaining at the midpoint of year $q-1$, and the capital investment remaining at the midpoint of year q, after accounting for all revenue payments at the original CRF rate c_1. This value is denoted by $K_q^{(1)}$. Having established the value of $K_q^{(1)}$, the updated CRF value c_2 is determined by the requirement that the present value of the after tax cash flows, corresponding to revenue payments after the CRF update, are equal to $K_q^{(1)}$.

In capital recovery year 1, the revenue is c_1K and the income tax payment is $(c_1K_0 - \delta_1K_0)s_1$. The repayment to investors at the midpoint of year 1 is the revenue net of taxes and the return on investment,

$$c_1K - (c_1K_0 - \delta_1K_0)s_1 - K_0\left(\sqrt{1+r_1} - 1\right).$$

K_0 is the investment principal, including debt and equity, at the start of the capital recovery period. The half year return on investment is given by $K_0\left(\sqrt{1+r_1} - 1\right)$. After restating the repayment to investors as

$$c_1K_0(1-s_1) + \delta_1K_0s_1 - K_0\left(\sqrt{1+r_1} - 1\right)$$

the investment remaining at the midpoint of year 1, denoted by K_1, is

$$\begin{aligned} K_1 &= K_0 - \left(c_1K_0(1-s_1) + \delta_1K_0s_1 - K_0\left(\sqrt{1+r_1} - 1\right)\right) \\ &= K_0\sqrt{1+r_1} - \mathrm{c}_1\mathrm{K}_0(1-s_1) - \delta_1\mathrm{K}_0\mathrm{s}_1. \end{aligned}$$

The revenue net of taxes and return on investment for year 2 is

$$\mathrm{c}_1\mathrm{K}_0(1-s_1) + \delta_2\mathrm{K}_0\mathrm{s}_1 - K_1r_1$$

where the return on investment ($K_1 r_1$) reflects a full year return at the initial after tax WACC rate. The equity investment remaining at the midpoint of year 2 is

$$K_2 = K_1 - (\mathrm{c}_1 \mathrm{K}_0(1 - s_1) + \delta_2 \mathrm{K}_0 \mathrm{s}_1 - K_1 r_1)$$

Restating K_2 as

$$K_2 = K_1(1 + r_1) - \mathrm{c}_1 \mathrm{K}_0(1 - s_1) - \delta_2 \mathrm{K}_0 \mathrm{s}_1$$

and replacing K_1 yields

$$K_2 = K_0(1 + r_1)^{3/2} - \mathrm{c}_1 \mathrm{K}_0(1 - s_1)[(1 + r_1) + 1] - \mathrm{K}_0 \mathrm{s}_1[\delta_1(1 + r_1) + \delta_2] .$$

Proceeding in this manner up to the year before the tax change, the equity investment remaining at the midpoint of year $m - 1$ is

$$K_{m-1} = K_0(1 + r_1)^{m-3/2} - \mathrm{c}_1 \mathrm{K}_0(1 - s_1) \sum_{j=0}^{m-2} (1 + r_1)^j - \mathrm{K}_0 \mathrm{s}_1 \sum_{j=1}^{m-1} \delta_\mathrm{j}(1 + r_1)^{m-j-1} .$$

Using equation (3.1)

$$\sum_{j=0}^{m-2} (1 + r_1)^j = \frac{(1 + r_1)^{m-1} - 1}{r_1}$$

and making the substitution yields

(2.2)

$$K_{m-1} = K_0 \left\{ (1 + r_1)^{m-3/2} - \mathrm{c}_1(1 - s_1) \frac{(1 + r_1)^{m-1} - 1}{r_1} - \mathrm{s}_1 \sum_{j=1}^{m-1} \delta_\mathrm{j}(1 + r_1)^{m-j-1} \right\} .$$

Case I-A: Tax rate changes on or before midpoint of year m ($\gamma \leq 0.5$)

The tax rate changes from s_1 to s_2 in year m. The net revenue calculation for year m and the expression for K_m must incorporate the tax rate change and a composite rate of return reflecting the two WACC rates r_1 and r_2. The revenue in year m will remain at $c_1 K_0$ since the CRF will not be changed until a later year under the current assumption that $m < q$. The tax rate is s_1 for the partial year γ and the tax rate is s_2 for the remaining $1 - \gamma$ portion of the year. The tax payment for year m is

$$\gamma(c_1K_0 - \delta_m K_0)s_1 + (1-\gamma)(c_1K_0 - \delta_m K_0)s_2$$

or equivalently

$$c_1K_0s_2 - \delta_m K_0 s_2 + \gamma(c_1 - \delta_m)K_0(s_1 - s_2).$$

The WACC rate changes when the tax rate changes and the investment return for year m is equal to the product of K_{m-1} and the rate of return given by $(1+r_1)^{.5+\gamma}(1+r_2)^{.5-\gamma} - 1$.[16] To align with the cases to come, it is helpful to rewrite the rate of return on investment as

$$\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_1) - 1\,.$$

The revenue net of taxes and return on investment for year m is

$$c_1K_0(1-s_2) + \delta_m K_0 s_2 - \gamma(c_1 - \delta_m)K_0(s_1 - s_2) - K_{m-1}\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_1) - 1\right]$$

and the investment remaining at the midpoint of year m, equal to K_{m-1} reduced by the year m revenue net of taxes and return on investment, is

$$K_m = K_{m-1}\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_1) - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2) - \delta_m K_0 s_2 + \gamma(c_1 - \delta_m)K_0(s_1 - s_2).$$

The revenue net of taxes and the return on investment for year $m+1$, in the case that $q > m+1$, is

$$c_1K_0(1-s_2) + \delta_{m+1}K_0s_2 - K_m r_2$$

and the investment remaining at the midpoint of year $m+1$ is

$$K_{m+1} = K_m(1+\mathrm{r}_2) - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2) - \delta_{\mathrm{m}+1}\mathrm{K}_0\mathrm{s}_2\,.$$

Upon replacing K_m,

16 The return calculation reflects the period from the midpoint of year $m-1$ to the midpoint of year m. Rate r_1 is applicable to the last half of year $m-1$ and a portion of year m of duration γ, which is assumed to be less than 0.5. Rate r_2 is applicable to the remaining time, $.5-\gamma$, up to the midpoint of year m.

$$K_{m+1} = K_{m-1}\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_1)(1+\mathrm{r}_2) - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)[(1+\mathrm{r}_2)+1]$$
$$- K_0 s_2[\delta_m(1+\mathrm{r}_2)+\delta_{\mathrm{m}+1}] + \gamma(c_1-\delta_m)K_0(s_1-s_2)(1+\mathrm{r}_2)$$

For year $m+2$, the revenue net of taxes and return on investment, in the case that $q > m+2$, is

$$c_1 K_0(1-s_2) + \delta_{m+2}K_0 s_2 - K_{m+1}r_2$$

and the capital investment remaining at the midpoint of year $m+2$ is

$$K_{m+2} = K_{m+1}(1+\mathrm{r}_2) - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2) - \delta_{\mathrm{m}+2}\mathrm{K}_0\mathrm{s}_2\,.$$

Upon replacing K_{m+1},

$$K_{m+2} = K_{m-1}\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_1)(1+\mathrm{r}_2)^2 - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)[(1+\mathrm{r}_2)^2+(1+\mathrm{r}_2)+1]$$
$$- K_0 s_2[\delta_m(1+\mathrm{r}_2)^2+\delta_{\mathrm{m}+1}(1+\mathrm{r}_2)+\delta_{\mathrm{m}+2}] + \gamma(c_1-\delta_m)K_0(s_1-s_2)(1+\mathrm{r}_2)^2\,.$$

Noting the pattern,

$$K_{q-1} = K_{m-1}\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_1)(1+\mathrm{r}_2)^{q-m-1} - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)\sum_{j=0}^{q-m-1}(1+\mathrm{r}_2)^j$$
$$- K_0 s_2\sum_{j=m}^{q-1}\delta_j(1+\mathrm{r}_2)^{q-1-j} + \gamma(c_1-\delta_m)K_0(s_1-s_2)(1+\mathrm{r}_2)^{q-m-1}\,.$$ [17]

Using (3.1)

$$\sum_{j=0}^{q-m-1}(1+\mathrm{r}_2)^j = \frac{(1+\mathrm{r}_2)^{q-m}-1}{\mathrm{r}_2}$$

and (2.2) to replace K_{m-1} yields

17 This expression follows by first recognizing $q-1 = m+(q-m-1)$.

(2.3)

$$K_{q-1} = K_0 \left\{ \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} (1+r_1)^{m-.5}(1+\mathrm{r}_2)^{q-m-1} \right.$$
$$- \mathrm{c}_1(1-s_1)\left(\frac{(1+r_1)^{m-1}-1}{r_1}\right)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} (1+r_1)(1+\mathrm{r}_2)^{q-m-1}$$
$$- \mathrm{s}_1\left(\sum_{j=1}^{m-1} \delta_{\mathrm{j}}(1+r_1)^{m-j-1}\right)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} (1+r_1)(1+\mathrm{r}_2)^{q-m-1}$$
$$- \mathrm{c}_1(1-\mathrm{s}_2)\left(\frac{(1+\mathrm{r}_2)^{q-m}-1}{\mathrm{r}_2}\right) - s_2\sum_{j=m}^{q-1} \delta_j(1+\mathrm{r}_2)^{q-1-j}$$
$$\left. + \gamma(c_1-\delta_m)(s_1-s_2)(1+\mathrm{r}_2)^{q-m-1} \right\}.$$

The CRF is updated in year q. The CRF c_1 determines the revenue for the partial year μ and the updated CRF c_2 determines the revenue for the remaining $1-\mu$ portion of the year. The revenue in year q, prior to the CRF update, net of taxes and the return on investment for year q is

$$\mu\left[c_1K_0 - \left(c_1K_0 - \delta_qK_0\right)s_2\right] - r_2K_{q-1}.$$

The investment remaining at the midpoint of service year q, after accounting for all revenue payments at the original CRF rate, is

(2.4)

$$K_q^{(1)} = K_{q-1}(1+r_2) - \mu c_1K_0(1-s_2) - \mu\delta_qK_os_2\,.$$

Replacing K_{q-1} with the expression in (2.3) yields

(2.5)

$$
\begin{aligned}
K_q^{(1)} = K_0 \Bigg\{ & \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} (1+r_1)^{m-.5}(1+\mathrm{r}_2)^{q-m} \\
& - \mathrm{c}_1(1-s_1)\left(\frac{(1+r_1)^{m-1}-1}{r_1}\right)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} (1+r_1)(1+\mathrm{r}_2)^{q-m} \\
& - \mathrm{s}_1\left(\sum_{j=1}^{m-1} \delta_\mathrm{j}(1+r_1)^{m-j-1}\right)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} (1+r_1)(1+\mathrm{r}_2)^{q-m} \\
& - \mathrm{c}_1(1-\mathrm{s}_2)\left(\frac{(1+\mathrm{r}_2)^{q-m}-1}{\mathrm{r}_2}\right)(1+r_2) - s_2\left(\sum_{j=m}^{q-1} \delta_j(1+\mathrm{r}_2)^{q-1-j}\right)(1+r_2) \\
& + \gamma(c_1-\delta_m)(s_1-s_2)(1+\mathrm{r}_2)^{q-m} - \mu c_1(1-s_2) - \mu\delta_q s_2 \Bigg\}.
\end{aligned}
$$

$K_q^{(1)}$ represents the remaining investment at the midpoint of year q after accounting for all revenue under the original CRF c_1.[18] An updated CRF c_2 is determined by requiring the present value of the future after tax cash flows is equal to $K_q^{(1)}$. The revenue after updating the CRF to c_2 in year q is $(1-\mu)c_2K_0$ and the after tax cash flow is the revenue less income taxes,

$$(1-\mu)\left[c_2K_0(1-s_2)+\delta_qK_0s_2\right].$$

The after tax cash flows for years $j = q+1$ through N are given by

$$c_2K_0(1-s_2)+\delta_jK_0s_2\,.$$

The present value of the after tax cash flows, at the midpoint of year q, is

(2.6)

$$(1-\mu)\left[c_2K_0(1-s_2)+\delta_qK_0s_2\right]+\sum_{j=q+1}^{N}\frac{c_2K_0(1-s_2)+\delta_jK_0s_2}{(1+r_2)^{j-q}}.$$

18 The superscript in the symbol, $K_q^{(1)}$, is used to recognize that this value does not represent the total investment remaining at the midpoint of year q. The after tax revenue attributable to the updated CRF c_2 in year q is not included.

The present value can be restated as

$$c_2K_0(1-s_2)(1+r_2)^q\sum_{j=q}^{N}\left(\frac{1}{1+r_2}\right)^j + K_0s_2\sum_{j=q}^{N}\frac{\delta_j}{(1+r_2)^{j-q}} - \mu c_2K_0(1-s_2) - \mu\delta_qK_0s_2$$

and then using (3.1) to replace the first summation,

(2.7)

$$c_2K_0(1-s_2)\left(\frac{(1+r_2)^{N-q+1}-1}{r_2(1+r_2)^{N-q}}\right) + K_0s_2\sum_{j=q}^{N}\frac{\delta_j}{(1+r_2)^{j-q}} - \mu c_2K_0(1-s_2) - \mu\delta_qK_0s_2\,.$$

If $q = N$, the expression for the present value of after tax cash flows in (2.7) reduces to

$$(1-\mu)[c_2K_0(1-s_2) + \delta_NK_0s_2]$$

which is the after tax cash flow after updating the CRF to c_2 in year N $(= q)$. This shows that (2.7) holds for $q \leq N$. Setting the present value of the after tax cash flows in (2.7) equal to $K_q^{(1)}$ and solving for c_2 yields

(2.8)

$$c_2 = \frac{r_2(1+r_2)^{N-q}}{(1-s_2)[(1+r_2)^{N-q+1} - 1 - \mu r_2(1+r_2)^{N-q}]}\left\{\frac{K_q^{(1)}}{K_0} - s_2\sum_{j=q}^{N}\frac{\delta_j}{(1+r_2)^{j-q}} + \mu\delta_qs_2\right\}.$$

The expression for c_2 in (2.1) is obtained from (2.8) by replacing $K_q^{(1)}$ with the expression in (2.5).

Case I-B: Tax rate changes in latter half of year m ($\gamma > 0.5$)

The income tax rate change occurs in the latter half ($\gamma > 0.5$) of year m. The investment principal remaining at the midpoint of year $m-1$ is the same as in Case I-A and the expression for K_{m-1} in (2.2) holds for Case I-B. The next step is to find K_m. The revenue and the income tax payment for year m are the same as in Case I-A. Since the WACC rate changes in the latter half of year m, the return on investment from the midpoint of year $m-1$ to the midpoint of year m is r_1K_{m-1} and the revenue net of taxes and return on investment for year m is

$$c_1K_0(1-s_2) + \delta_mK_0s_2 - \gamma(c_1-\delta_m)K_0(s_1-s_2) - r_1K_{m-1}\,.$$

The investment remaining at the midpoint of year m, equal to K_{m-1} reduced by the year m revenue net of taxes and return on investment is

$$K_m = K_{m-1}(1 + r_1) - \mathrm{c}_1\mathrm{K}_0(1 - \mathrm{s}_2) - \delta_m K_0 s_2 + \gamma(c_1 - \delta_m)K_0(s_1 - s_2).$$

The WACC rate changes at some point between the midpoint of year m and the end of year m. The rate of return on investment over the period from the midpoint of year m to the midpoint of year $m + 1$ is $(1 + r_1)^{\gamma-.5}(1 + r_2)^{1.5-\gamma} - 1$.[19] To align with other cases it is helpful to rewrite the rate of return on investment as

$$\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5} (1 + r_2) - 1.$$

The revenue net of taxes and return on investment for year $m + 1$ is

$$c_1 K_0(1 - s_2) + \delta_{m+1} K_0 s_2 - K_m \left[\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5} (1 + r_2) - 1\right]$$

and the investment remaining at the midpoint of year $m + 1$ is

$$K_{m+1} = K_m \left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5} (1 + r_2) - \mathrm{c}_1\mathrm{K}_0(1 - \mathrm{s}_2) - \delta_{\mathrm{m}+1}\mathrm{K}_0\mathrm{s}_2\,.$$

Replacing K_m yields

$$\begin{aligned} K_{m+1} = K_{m-1} &\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5} (1 + r_1)(1 + r_2) - \mathrm{c}_1\mathrm{K}_0(1 - \mathrm{s}_2)\left[\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5} (1 + r_2) + 1\right] \\ &- K_0 s_2\left[\delta_m \left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5} (1 + r_2) + \delta_{\mathrm{m}+1}\right] \\ &+ \gamma(c_1 - \delta_m)K_0(s_1 - s_2)\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5} (1 + r_2)\,. \end{aligned}$$

Using the observations that

$$\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5} (1 + r_2) + 1 = \left[\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5} - 1\right](1 + r_2) + [(1 + r_2) + 1]$$

[19] Rate r_1 is applicable from the midpoint of year m to the point of the income tax change which has duration $\gamma - .5$ under the assumption $\gamma > 0.5$. Rate r_2 is applicable to the remaining portion of year m, $1 - \gamma$, plus the first half of year $m + 1$.

and

$$\delta_m \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)+\delta_{\mathrm{m+1}} = \delta_m\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)+\delta_m(1+r_2)+\delta_{\mathrm{m+1}}$$

leads to the following restatement of K_{m+1}

$$\begin{aligned}K_{m+1} = K_{m-1}\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}&(1+r_1)(1+r_2)-\mathrm{c_1K_0(1-s_2)}[(1+r_2)+1]\\&-K_0s_2[\delta_m(1+r_2)+\delta_{\mathrm{m+1}}]+\gamma(c_1-\delta_m)K_0(s_1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)\\&-\mathrm{c_1K_0(1-s_2)}\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)-K_0s_2\delta_m\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)\,.\end{aligned}$$ [20]

For year $m+2$, the revenue net of taxes and return on investment, in the case that $q > m+2$, is

$$c_1K_0(1-s_2)+\delta_{m+2}K_0s_2-K_{m+1}r_2$$

and the capital investment remaining at the midpoint of year $m+2$ is

$$K_{m+2} = K_{m+1}(1+\mathrm{r_2})-\mathrm{c_1K_0(1-s_2)}-\delta_{\mathrm{m+2}}\mathrm{K_0s_2}\,.$$

Upon replacing K_{m+1}

$$\begin{aligned}K_{m+2} = K_{m-1}\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}&(1+r_1)(1+r_2)^2-\mathrm{c_1K_0(1-s_2)}[(1+r_2)^2+(1+r_2)+1]\\&-K_0s_2[\delta_m(1+r_2)^2+\delta_{\mathrm{m+1}}(1+r_2)+\delta_{\mathrm{m+2}}]\\&+\gamma(c_1-\delta_m)K_0(s_1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)^2\\&-\mathrm{c_1K_0(1-s_2)}\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^2-K_0s_2\delta_m\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^2\,.\end{aligned}$$

Noting the pattern,

[20] This algebraic manipulation, while not necessary for the derivation of this subcase in isolation, will lead to an expression for c_2 that aligns with the general expression in (2.1)

$$K_{q-1} = K_{m-1}\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_1)(1+r_2)^{q-m-1} - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)\sum_{j=0}^{q-m-1}(1+r_2)^j$$
$$- K_0 s_2 \sum_{j=m}^{q-1}\delta_j(1+r_2)^{q-1-j} + \gamma(c_1-\delta_m)K_0(s_1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)^{q-m-1}$$
$$- \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right](1+r_2)^{q-m-1}$$
$$- K_0 s_2 \delta_m\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right](1+r_2)^{q-m-1}.$$ [21]

Using (3.1)

$$\sum_{j=0}^{q-m-1}(1+\mathrm{r}_2)^j = \frac{(1+\mathrm{r}_2)^{q-m}-1}{\mathrm{r}_2}$$

and (2.2) to replace K_{m-1} yields

[21] This expression follows by first recognizing $q-1 = m + (q-m-1)$.

(2.9)

$$
\begin{aligned}
K_{q-1} = K_0 \Bigg\{ & \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} (1+r_1)^{m-.5}(1+r_2)^{q-m-1} \\
& - \mathrm{c}_1(1-s_1)\left(\frac{(1+r_1)^{m-1}-1}{r_1}\right)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_1)(1+r_2)^{q-m-1} \\
& - \mathrm{s}_1\left(\sum_{j=1}^{m-1} \updelta_{\mathrm{j}}(1+r_1)^{m-j-1}\right)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_1)(1+r_2)^{q-m-1} \\
& - \mathrm{c}_1(1-\mathrm{s}_2)\left(\frac{(1+\mathrm{r}_2)^{q-m}-1}{\mathrm{r}_2}\right) - s_2\sum_{j=m}^{q-1}\delta_j(1+r_2)^{q-1-j} \\
& + \gamma(c_1-\delta_m)(s_1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)^{q-m-1} \\
& - \mathrm{c}_1(1-\mathrm{s}_2)\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^{q-m-1} \\
& - s_2\delta_m\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^{q-m-1}\Bigg\}.
\end{aligned}
$$

The remainder of the derivation for this subcase is the same as Case I-A. The expression for $K_q^{(1)}$ in (2.4) holds for this case and upon replacing K_{q-1} using (2.9),

(2.10)

$$K_q^{(1)} = K_0 \left\{ \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} (1+r_1)^{m-.5}(1+r_2)^{q-m} \right.$$

$$- \mathrm{c}_1(1-s_1)\left(\frac{(1+r_1)^{m-1}-1}{r_1}\right)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} (1+r_1)(1+r_2)^{q-m}$$

$$- \mathrm{s}_1\left(\sum_{j=1}^{m-1} \delta_{\mathrm{j}}(1+r_1)^{m-j-1}\right)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} (1+r_1)(1+r_2)^{q-m}$$

$$- \mathrm{c}_1(1-\mathrm{s}_2)\left(\frac{(1+\mathrm{r}_2)^{q-m}-1}{\mathrm{r}_2}\right)(1+r_2) - s_2\left(\sum_{j=m}^{q-1} \delta_j(1+r_2)^{q-1-j}\right)(1+r_2)$$

$$+ \gamma(c_1-\delta_m)(s_1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} (1+r_2)^{q-m}$$

$$- \mathrm{c}_1(1-\mathrm{s}_2)\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right](1+r_2)^{q-m} - s_2\delta_m\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right](1+r_2)^{q-m}$$

$$\left. - \mu c_1(1-s_2) - \mu\delta_q s_2 \right\}.$$

Equations (2.7) and (2.8) hold for this case and the expression for c_2 in (2.1) is obtained from (2.8) by replacing $K_q^{(1)}$ with the expression in (2.10).

2.3.2 Case II: Tax rate change and CRF update occur in same year, after the first year ($\mathbf{1 < m = q \leq N}$)

In Case II, the change in the income tax rate and the CRF update occur in the same capital recovery year ($m = q$). The expression for K_{m-1} in (2.2) established in Case I-A holds for Case II.

Case II-A: Tax rate changes on or before midpoint of year m ($\gamma \leq 0.5$)

Since $K_{q-1} = K_{m-1}$ Case II, the next step is to derive an expression for $K_q^{(1)}$, the investment remaining at the midpoint of service year q, after accounting for all revenue payments prior to the CRF update. The revenue for year q at the original CRF c_1 is $\mu c_1 K_0$ and the income tax for this portion of the revenue is

$$\gamma\big(c_1 K_0 - \delta_q K_0\big)s_1 + (\mu - \gamma)\big(c_1 K_0 - \delta_q K_0\big)s_2$$

which can be restated as

$$\gamma\big(c_1 - \delta_q\big)K_0(s_1 - s_2) + \mu c_1 K_0 s_2 - \mu \delta_q K_0 s_2\,.$$

The rate of return on investment for year m (= q) reflects the change in the WACC rate and is given by

$$\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma - .5}(1 + r_1) - 1.$$

The revenue in year q, prior to the CRF update, net income taxes and the return on investment is

$$\mu c_1 K_0(1 - s_2) - \gamma\big(c_1 - \delta_q\big)K_0(s_1 - s_2) + \mu \delta_q K_0 s_2 - K_{m-1}\left[\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma - .5}(1 + r_1) - 1\right]$$

and the investment remaining at the midpoint of service year q, after accounting for all revenue payments under the original CRF c_1 is

$$K_q^{(1)} = K_{m-1}\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma - .5}(1 + r_1) + \gamma\big(c_1 - \delta_q\big)K_0(s_1 - s_2) - \mu c_1 K_0(1 - s_2) - \mu \delta_q K_0 s_2\,.$$

Using (2.2) to replace K_{m-1} yields

(2.11)

$$\begin{aligned} K_q^{(1)} = K_0 \Bigg\{ & \left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma - .5}(1 + r_1)^{m - .5} - \mathrm{c}_1(1 - s_1)\left(\frac{(1 + r_1)^{m-1} - 1}{r_1}\right)\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma - .5}(1 + r_1) \\ & - \mathrm{s}_1\left(\sum_{j=1}^{m-1} \delta_{\mathrm{j}}(1 + r_1)^{m-j-1}\right)\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma - .5}(1 + r_1) + \gamma\big(c_1 - \delta_q\big)(s_1 - s_2) \\ & - \mu c_1(1 - s_2) - \mu \delta_q s_2 \Bigg\}. \end{aligned}$$

The revenue payments after updating the CRF to c_2 in year q and the corresponding after tax cash flows and discount rates are the same as in Case I. Therefore (2.7) and (2.8) hold for this case and the expression for c_2 in (2.1) is obtained from (2.8) by replacing $K_q^{(1)}$ with the expression in (2.11).

Case II-B – Tax rate changes in latter half of year m ($\gamma > 0.5$)

The revenue for year q at the original CRF c_1 is $\mu c_1 K_0$ and corresponding income tax is

$$\gamma(c_1 - \delta_q)K_0(s_1 - s_2) + \mu c_1 K_0 s_2 - \mu \delta_q K_0 s_2 \,.$$ [22]

The rate of return on investment for year m $(= q)$ is r_1 since the WACC rate does not change until the latter half of year m. The revenue in year q, prior to the CRF update, net income taxes and the return on investment is

$$\mu c_1 K_0(1 - s_2) - \gamma(c_1 - \delta_q)K_0(s_1 - s_2) + \mu \delta_q K_0 s_2 - r_1 K_{m-1}$$

and the investment remaining at the midpoint of service year q, after accounting for all revenue payments under the original CRF c_1 is

$$K_q^{(1)} = K_{m-1}(1 + r_1) + \gamma(c_1 - \delta_q)K_0(s_1 - s_2) - \mu c_1 K_0(1 - s_2) - \mu \delta_q K_0 s_2 \,.$$

Using (2.2) to replace K_{m-1} yields

(2.12)

$$K_q^{(1)} = K_0 \Bigg\{ (1 + r_1)^{m-.5} - \mathrm{c}_1(1 - s_1)\left(\frac{(1 + r_1)^{m-1} - 1}{r_1}\right)(1 + r_1) - \mathrm{s}_1\left(\sum_{j=1}^{m-1} \delta_\mathrm{j}(1 + r_1)^{m-j-1}\right)(1 + r_1) + \gamma(c_1 - \delta_q)(s_1 - s_2) - \mu c_1(1 - s_2) - \mu \delta_q s_2 \Bigg\}.$$

The updated CRF c_2 is determined by the requirement that the present value of the after tax cash flows, from revenue payments after the CRF update, is equal to $K_q^{(1)}$. The revenue after updating the CRF to c_2 in year q is $(1 - \mu)c_2 K_0$ and the corresponding after tax cash flow is

$$(1 - \mu)\left[c_2 K_0(1 - s_2) + \delta_q K_0 s_2\right].$$

The after tax cash flows for years $j = q + 1$ through N are

$$c_2 K_0(1 - s_2) + \delta_j K_0 s_2 \,.$$

[22] Same as in Case II-A.

This case differs from previous cases due to the discount rate for the period from the midpoint of year q to the midpoint of year $q+1$. Because the income tax change occurred in the latter half of year q the discount rate is $(1+r_1)^{.5-\gamma}(1+r_2)^{1.5-\gamma}-1$ or equivalently

$$\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)-1.$$

The corresponding discount factor is

$$\frac{1}{1+\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)-1\right]}$$

or equivalently

$$\left(\frac{1+r_2}{1+r_1}\right)^{\gamma-.5}\left(\frac{1}{1+r_2}\right).$$

The discount rate for all other years in the present value calculation is r_2 and the present value of the after tax cash flows, at the midpoint of year q, is

(2.13)

$$(1-\mu)\left[c_2K_0(1-s_2)+\delta_qK_0s_2\right]+\left(\frac{1+r_2}{1+r_1}\right)^{\gamma-.5}\frac{1}{1+r_2}\sum_{j=q+1}^{N}\frac{c_2K_0(1-s_2)+\delta_jK_0s_2}{(1+r_2)^{j-q-1}}$$

which can be restated as

$$\left(\frac{1+r_2}{1+r_1}\right)^{\gamma-.5}\left\{c_2K_0(1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}+c_2K_0(1-s_2)\sum_{j=q+1}^{N}\frac{1}{(1+r_2)^{j-q}}+\delta_qK_0s_2\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\right.$$
$$\left.+K_0s_2\sum_{j=q+1}^{N}\frac{\delta_j}{(1+r_2)^{j-q}}-\mu c_2K_0(1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-\mu\delta_qK_0s_2\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\right\}.$$

Additional algebraic manipulation is necessary to match the general formula in (2.1). Using the observations

$$\sum_{j=q+1}^{N}\frac{1}{(1+r_2)^{j-q}}=\sum_{j=q}^{N}\frac{1}{(1+r_2)^{j-q}}-1$$

and

$$\sum_{j=q+1}^{N} \frac{\delta_j}{(1+r_2)^{j-q}} = \sum_{j=q}^{N} \frac{\delta_j}{(1+r_2)^{j-q}} - \delta_q$$

the present value of after tax cash flows can be restated as

$$\left(\frac{1+r_2}{1+r_1}\right)^{\gamma-.5} \left\{ c_2 K_0 (1-s_2) \sum_{j=q}^{N} \frac{1}{(1+r_2)^{j-q}} + c_2 K_0 (1-s_2) \left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right] + K_0 s_2 \sum_{j=q}^{N} \frac{\delta_j}{(1+r_2)^{j-q}} \right.$$
$$\left. + \delta_q K_0 s_2 \left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right] - \mu c_2 K_0 (1-s_2) \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - \mu \delta_q K_0 s_2 \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} \right\}.$$

Using (3.1)

$$\sum_{j=q}^{N} \frac{1}{(1+r_2)^{j-q}} = (1+r_2)^q \sum_{j=q}^{N} \left(\frac{1}{1+r_2}\right)^j = \frac{(1+r_2)^{N-q+1} - 1}{r_2 (1+r_2)^{N-q}}$$

and after making the substitution the present value of the after tax cash flows is

(2.14)

$$\left(\frac{1+r_2}{1+r_1}\right)^{\gamma-.5} \left\{ c_2 K_0 (1-s_2) \left(\frac{(1+r_2)^{N-q+1} - 1}{r_2 (1+r_2)^{N-q}}\right) + c_2 K_0 (1-s_2) \left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right] \right.$$
$$+ K_0 s_2 \sum_{j=q}^{N} \frac{\delta_j}{(1+r_2)^{j-q}} + \delta_q K_0 s_2 \left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right] - \mu c_2 K_0 (1-s_2) \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}$$
$$\left. - \mu \delta_q K_0 s_2 \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} \right\}.$$

If $q = N$ the expression for the present value of after cash tax flows in (2.14) reduces to the after tax cash flow for year N

$$(1-\mu)[c_2 K_0 (1-s_2) + \delta_N K_0 s_2]$$

and therefore (2.14) holds for $q \leq N$. Setting $K_q^{(1)}$ equal to the present value of after tax cash flows in (2.14) and solving for c_2 yields

(2.15)

$$c_2 = \frac{r_2(1+r_2)^{N-q}}{(1-s_2)\left\{(1+r_2)^{N-q+1} - 1 - \mu\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} r_2(1+r_2)^{N-q} + \left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right] r_2(1+r_2)^{N-q}\right\}}$$
$$\cdot \left\{\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} \frac{K_q^{(1)}}{K_0} - \delta_q s_2 \left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right] + \mu \delta_q s_2 \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} \right.$$
$$\left. - s_2 \sum_{j=q}^{N} \frac{\delta_j}{(1+r_2)^{j-q}} \right\}.$$

The expression for c_2 in (2.1) is obtained from (2.15) by replacing $K_q^{(1)}$ with the expression in (2.12).

2.3.3 Case III: Tax rate change occurs in the first year, CRF is updated in a later year ($1 = m < q \leq N$)

The income tax rate change and the associated change in the after tax WACC rate occurs in the first year of the capital recovery term. The income tax payment in year 1 is

$$\gamma(c_1K_0 - \delta_1K_0)s_1 + (1-\gamma)(c_1K_0 - \delta_1K_0)s_2$$

or equivalently

$$c_1K_0s_2 - \delta_1K_0s_2 + \gamma(c_1 - \delta_1)K_0(s_1 - s_2).$$

Case III-A: Tax rate changes on or before midpoint of year 1 ($\gamma \leq 0.5$)

In Case III-A, the after tax WACC rate changes from r_1 to r_2 during the first half of the year and the rate of return on investment for first half of recovery year 1 is

$$(1+r_1)^{\gamma}(1+r_2)^{.5-\gamma} - 1$$

or equivalently

$$\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} \sqrt{1+r_1} - 1\,.$$

The revenue net the income tax payment and the return on investment for year 1 is

$$c_1K_0(1-s_2)+\delta_1K_0s_2-(c_1-\delta_1)K_0(s_1-s_2)-K_0\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\sqrt{1+r_1}-1\right]$$

where the return on investment is the product of the initial capital investment (K_0) and half year rate of return. The investment remaining at the midpoint of year 1 is

$$K_1=K_0\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\sqrt{1+r_1}-\mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)-\delta_1K_0s_2+\gamma(c_1-\delta_1)K_0(s_1-s_2).$$

The new tax rate s_2 is effective for year 2. The revenue net of taxes and return on investment for year 2, in the case that $q>2$, is

$$c_1K_0(1-s_2)+\delta_2K_0s_2-K_1r_2$$

and the capital investment remaining at the midpoint of year 2 is

$$K_2=K_1(1+\mathrm{r}_2)-\mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)-\delta_2\mathrm{K}_0\mathrm{s}_2\,.$$

After replacing K_1,

$$K_2=K_0\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\sqrt{1+r_1}(1+\mathrm{r}_2)-\mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)[(1+\mathrm{r}_2)+1]-K_0s_2[\delta_1(1+\mathrm{r}_2)+\delta_2]$$
$$+\gamma(c_1-\delta_1)K_0(s_1-s_2)(1+\mathrm{r}_2)\,.$$

Noting the pattern,

$$K_{q-1}=K_0\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\sqrt{1+r_1}(1+\mathrm{r}_2)^{q-2}-\mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)\sum_{j=1}^{q-1}(1+\mathrm{r}_2)^{j-1}$$
$$-K_0s_2\sum_{j=1}^{q-1}\delta_j(1+\mathrm{r}_2)^{q-1-j}+\gamma(c_1-\delta_1)K_0(s_1-s_2)(1+\mathrm{r}_2)^{q-2}\,.$$

Using equation (3.1) yields

$$\sum_{j=1}^{q-1}(1+\mathrm{r}_2)^{j-1}=\frac{(1+\mathrm{r}_2)^{q-1}-1}{\mathrm{r}_2}\,.$$ [23]

Upon replacing the summation

23 Restate the summation as $(1+\mathrm{r}_2)^{-1}\sum_{j=1}^{q-1}(1+\mathrm{r}_2)^j$ and then use (3.1).

(2.16)

$$K_{q-1} = K_0\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\sqrt{1+r_1}(1+\mathrm{r}_2)^{q-2} - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)\left(\frac{(1+\mathrm{r}_2)^{q-1}-1}{\mathrm{r}_2}\right)$$
$$- K_0 s_2\sum_{j=1}^{q-1}\delta_j(1+\mathrm{r}_2)^{q-1-j} + \gamma(c_1-\delta_1)K_0(s_1-s_2)(1+\mathrm{r}_2)^{q-2}\,.$$

In year q the CRF is updated. From this point forward the derivations proceeds as in Case I-A. Equation (2.4) holds for this case and replacing K_{q-1} in (2.4) with the expression in (2.16) yields

(2.17)

$$K_q^{(1)} = K_0\Bigg\{\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\sqrt{1+r_1}(1+\mathrm{r}_2)^{q-1} - \mathrm{c}_1(1-\mathrm{s}_2)\left(\frac{(1+\mathrm{r}_2)^{q-1}-1}{\mathrm{r}_2}\right)(1+r_2)$$
$$- s_2\left(\sum_{j=1}^{q-1}\delta_j(1+\mathrm{r}_2)^{q-1-j}\right)(1+r_2) + \gamma(c_1-\delta_1)(s_1-s_2)(1+\mathrm{r}_2)^{q-1} - \mu c_1(1-s_2)$$
$$- \mu\delta_q s_2\Bigg\}.$$

The updated CRF c_2 is determined by requiring that the present value of the after tax cash flows, after the CRF update, is equal to $K_q^{(1)}$. The after tax cash flows and the corresponding discount rates for this subcase are the same as in Case I and therefore (2.7) and (2.8) hold for this subcase. The expression for c_2 in (2.1) is obtained by replacing $K_q^{(1)}$ in (2.8) with the expression in (2.17)**Error! Reference source not found.**.

Case III-B: Tax rate changes in latter half of year 1 ($\gamma > 0.5$)

As in Case III-A, the income tax payment for year 1 is

$$c_1K_0s_2 - \delta_1K_0s_2 + \gamma(c_1-\delta_1)K_0(s_1-s_2)\,.$$

The WACC rate changes in the latter half of year 1, the half year return on investment at the midpoint of year 1 is $K_0\left(\sqrt{1+r_1}-1\right)$ and the revenue net of taxes and return on investment for year 1 is

$$c_1K_0(1-s_2) + \delta_1K_0s_2 - \gamma(c_1-\delta_1)K_0(s_1-s_2) - K_0\left(\sqrt{1+r_1}-1\right).$$

The investment remaining at the midpoint of year 1, equal to the initial investment K_0 reduced by the year 1 revenue net of taxes and return on investment is

$$K_1 = K_0\sqrt{1+r_1} - \mathrm{c_1 K_0 (1 - s_2)} - \delta_1 K_0 s_2 + \gamma(c_1 - \delta_1) K_0 (s_1 - s_2).$$

The rate of return on investment over the period from the midpoint of year 1 to the midpoint of year 2 is $(1+r_1)^{\gamma-.5}(1+r_2)^{1.5-\gamma} - 1$, or equivalently

$$\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2) - 1\,.$$[24]

The revenue net of taxes and return on investment for year 2 is

$$c_1 K_0 (1 - s_2) + \delta_2 K_0 s_2 - K_1\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2) - 1\right]$$

and the investment remaining at the midpoint of year 2 is

$$K_2 = K_1\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2) - \mathrm{c_1 K_0 (1 - s_2)} - \mathrm{\delta_2 K_0 s_2}\,.$$

Replacing K_1 yields

$$\begin{aligned} K_2 = K_0\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}&\sqrt{1+r_1}(1+r_2) - \mathrm{c_1 K_0 (1 - s_2)}\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2) + 1\right] \\ &- K_0 s_2\left[\delta_1\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2) + \delta_2\right] + \gamma(c_1 - \delta_1)K_0(s_1 - s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2) \end{aligned}$$

Using the observations that

$$\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2) + 1 = \left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right](1+r_2) + (1+r_2) + 1$$

and

$$\delta_1\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2) + \delta_2 = \delta_1\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right](1+r_2) + \delta_1(1+r_2) + \delta_2$$

---

24 Rate r_1 is applicable from the midpoint of year 1 to the point of the income tax change which has duration $\gamma - .5$ under the assumption $\gamma > 0.5$. Rate r_2 is applicable to the remaining portion of year 1, $1 - \gamma$, plus the first half of year 2.

leads to the following restatement of K_2

$$K_2 = K_0 \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} \sqrt{1+r_1}(1+r_2) - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)[(1+r_2)+1] - K_0 s_2[\delta_1(1+r_2)+\delta_2]$$
$$+ \gamma(c_1-\delta_1)K_0(s_1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)$$
$$- \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2) - K_0 s_2 \delta_1 \left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)\,.$$

For year 3, the revenue net of taxes and return on investment, in the case that $q > 3$, is

$$c_1 K_0(1-s_2) + \delta_3 K_0 s_2 - K_2 r_2$$

and the capital investment remaining at the midpoint of year 3 is

$$K_3 = K_2(1+\mathrm{r}_2) - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2) - \delta_3\mathrm{K}_0\mathrm{s}_2\,.$$

Upon replacing K_2

$$K_3 = K_0\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\sqrt{1+r_1}(1+r_2)^2 - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)[(1+r_2)^2+(1+r_2)+1]$$
$$- K_0 s_2[\delta_1(1+r_2)^2 + \delta_2(1+r_2)+\delta_3]$$
$$+ \gamma(c_1-\delta_1)K_0(s_1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)^2$$
$$- \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^2 - K_0 s_2\delta_1\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^2$$

Noting the pattern,

$$K_{q-1} = K_0\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\sqrt{1+r_1}(1+r_2)^{q-2} - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)\sum_{j=0}^{q-2}(1+r_2)^j - K_0 s_2\sum_{j=1}^{q-1}\delta_j(1+r_2)^{q-1-j}$$
$$+ \gamma(c_1-\delta_1)K_0(s_1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)^{q-2}$$
$$- \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^{q-2}$$
$$- K_0 s_2\delta_1\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^{q-2}$$

Using (3.1)

$$\sum_{j=0}^{q-2}(1+\mathrm{r}_2)^j = \frac{(1+\mathrm{r}_2)^{q-1}-1}{\mathrm{r}_2}$$

and upon making the substitution

(2.18)

$$\begin{aligned}K_{q-1} = {} & K_0\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\sqrt{1+r_1}(1+\mathrm{r}_2)^{q-2} - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)\left(\frac{(1+\mathrm{r}_2)^{q-1}-1}{\mathrm{r}_2}\right) \\ & - K_0 s_2\sum_{j=1}^{q-1}\delta_j(1+r_2)^{q-1-j} + \gamma(c_1-\delta_1)K_0(s_1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)^{q-2} \\ & - \mathrm{c}_1\mathrm{K}_0(1-\mathrm{s}_2)\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^{q-2} \\ & - K_0 s_2\delta_1\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^{q-2}\end{aligned}$$

The remainder of the derivation proceeds as in Case III-A. The expression for $K_q^{(1)}$ in (2.4) holds for this case and using (2.18) to replace K_{q-1} yields

(2.19)

$$\begin{aligned}K_q^{(1)} = K_0\Bigg\{ & \left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\sqrt{1+r_1}(1+\mathrm{r}_2)^{q-1} - \mathrm{c}_1(1-\mathrm{s}_2)\left(\frac{(1+\mathrm{r}_2)^{q-1}-1}{\mathrm{r}_2}\right)(1+r_2) \\ & - s_2\left(\sum_{j=1}^{q-1}\delta_j(1+r_2)^{q-1-j}\right)(1+r_2) + \gamma(c_1-\delta_1)(s_1-s_2)\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2)^{q-1} \\ & - \mathrm{c}_1(1-\mathrm{s}_2)\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^{q-1} - s_2\delta_1\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}-1\right](1+r_2)^{q-1} \\ & - \mu c_1(1-s_2) - \mu\delta_q s_2\Bigg\}\end{aligned}$$

The updated CRF c_2 is determined by requiring that the present value of the after tax cash flows, after the CRF update, is equal to $K_q^{(1)}$. The after tax cash flows and the corresponding discount rates for this subcase are the same as in Case I and therefore (2.7) and (2.8) hold for this subcase. The expression for c_2 in (2.1) is obtained from (2.8) by replacing $K_q^{(1)}$ with the expression in (2.19).

2.3.4 Case IV: Tax rate change and CRF Update in first year ($1 = m = q \leq N$)

In Case IV the income tax change occurs in the first year of capital recovery and the CRF is also updated in the first year of capital recovery. The year 1 revenue from payments at the original CRF are equal to $\mu c_1 K_0$ and the associated income tax is

$$\gamma(c_1 K_0 - \delta_1 K_0)s_1 + (\mu - \gamma)(c_1 K_0 - \delta_1 K_0)s_2$$

which can be restated as

$$\mu c_1 K_0 s_2 + \gamma(c_1 - \delta_1)K_0(s_1 - s_2) - \mu\delta_1 K_0 s_2 \,.$$

Case IV-A: Tax rate changes on or before midpoint of year 1 ($\gamma \leq 0.5$)

The after tax WACC rate changes from r_1 to r_2 during the first half of year 1and the rate of return on investment for first half of recovery year 1 is

$$(1 + r_1)^{\gamma}(1 + r_2)^{.5-\gamma} - 1$$

or equivalently

$$\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5}\sqrt{1 + r_1} - 1\,.$$

The revenue corresponding to payments at the original CRF c_1 net the corresponding income tax payment and the return on investment for year 1 is

$$\mu c_1 K_0 - [\mu c_1 K_0 s_2 + \gamma(c_1 - \delta_1)K_0(s_1 - s_2) - \mu\delta_1 K_0 s_2] - K_0\left[\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5}\sqrt{1 + r_1} - 1\right]$$

where the return on investment is the product of the initial capital investment (K_0) and the half year rate of return. The investment remaining at the midpoint of year 1, after accounting for all revenue payments at the original CRF rate c_1, is

(2.20)

$$K_1^{(1)} = K_0\left\{\left(\frac{1 + r_1}{1 + r_2}\right)^{\gamma-.5}\sqrt{1 + r_1} - \mu c_1(1 - s_2) + \gamma(c_1 - \delta_1)(s_1 - s_2) - \mu\delta_1 s_2\right\}.$$

The updated CRF c_2 is determined by requiring the present value of the after tax cash flows, associated with revenue payments at the updated CRF rate, is equal to $K_1^{(1)}$. In the first year, the revenue at the updated CRF rate is $(1-\mu)c_2K_0$. The corresponding income tax payment is $(1-\mu)[c_2K_0 - \delta_1K_0]s_2$ and the after tax cash flow for year 1 is

$$(1-\mu)[c_2K_0(1-s_2) + \delta_1K_0s_2]\,.$$

The after tax cash flows for years $j = 2$ through N are

$$c_2K_0(1-s_2) + \delta_jK_0s_2\,.$$

Because the income tax rate change occurs on or before the midpoint of year 1, under the assumptions of this subcase, the discount rate is r_2 for all years in the present value calculation. The present value of the after tax cash flows, associated with revenue payments at the updated CRF rate, is

$$(1-\mu)[c_2K_0(1-s_2) + \delta_1K_0s_2] + \sum_{j=2}^{N}\frac{c_2K_0(1-s_2) + \delta_jK_0s_2}{(1+r_2)^{j-1}}$$

which can be restated as

$$c_2K_0(1-s_2)\sum_{j=1}^{N}\left(\frac{1}{1+r_2}\right)^{j-1} + K_0s_2\sum_{j=1}^{N}\frac{\delta_j}{(1+r_2)^{j-1}} - \mu c_2K_0(1-s_2) - \mu\delta_1K_0s_2\,.$$

Using (3.1),

(2.21)

$$\sum_{j=1}^{N}\left(\frac{1}{1+r_2}\right)^{j-1} = \frac{(1+r_2)^N - 1}{r_2(1+r_2)^{N-1}}$$

and making the substitution yields

(2.22)

$$c_2K_0(1-s_2)\left(\frac{(1+r_2)^N - 1}{r_2(1+r_2)^{N-1}}\right) + K_0s_2\sum_{j=1}^{N}\frac{\delta_j}{(1+r_2)^{j-1}} - \mu c_2K_0(1-s_2) - \mu\delta_1K_0s_2\,.$$

If $N = 1$, the expression in (2.22) reduces to the after tax cash flow in the first year and therefore (2.22) holds for $N \geq 1$.

Setting the present value of after tax cash flows in (2.22) equal to $K_1^{(1)}$ and solving for c_2 yields

(2.23)

$$c_2 = \left(\frac{r_2(1+r_2)^{N-1}}{(1-s_2)[(1+r_2)^N - 1 - \mu r_2(1+r_2)^{N-1}]}\right)\left\{\frac{K_1^{(1)}}{K_0} - s_2 \sum_{j=1}^{N} \frac{\delta_j}{(1+r_2)^{j-1}} + \mu\delta_1 s_2\right\}.$$

The expression for c_2 in (2.1) is obtained by replacing $K_1^{(1)}$ in (2.23) with the expression in (2.20).

Case IV-B: Tax rate changes in latter half of year 1 ($\gamma > 0.5$)

The year 1 revenue from payments at the original CRF and the associated income tax are the same as in Case IV-A. In Case IV-B, the after tax WACC rate changes from r_1 to r_2 during the latter half of year 1 and the rate of return on investment for the first half of recovery year 1 is $\sqrt{1+r_1} - 1$. The revenue, corresponding to payments at the original CRF rate c_1, net the corresponding income tax payment and the return on investment for year 1 is

$$\mu c_1 K_0 - [\mu c_1 K_0 s_2 + \gamma(c_1 - \delta_1)K_0(s_1 - s_2) - \mu\delta_1 K_0 s_2] - K_0\left(\sqrt{1+r_1} - 1\right)$$

where the return on investment is the product of the initial capital investment (K_0) and the half year rate of return. The investment remaining at the midpoint of year 1, after accounting for all revenue payments prior to the CRF update, is

(2.24)

$$K_1^{(1)} = K_0\left\{\sqrt{1+r_1} - \mu c_1(1-s_2) + \gamma(c_1 - \delta_1)(s_1 - s_2) - \mu\delta_1 s_2\right\}.$$

The updated CRF c_2 is determined by requiring the present value of the after tax cash flows, associated with revenue payments at the updated CRF rate, is equal to $K_1^{(1)}$. As in Case IV-A, the after tax cash flow in the first year, corresponding to revenue payments at the updated CRF rate, is

$$(1-\mu)[c_2 K_0(1-s_2) + \delta_1 K_0 s_2]$$

and the after tax cash flows for years $j = 2$ through N are

$$c_2 K_0(1 - s_2) + \delta_j K_0 s_2 \,.$$

Because the income tax rate change occurs during the latter half of year 1, under the assumptions of this subcase, the discount rate for the period from the midpoint of year 1 to the midpoint of year 2 is $(1+r_1)^{\gamma-.5}(1+r_2)^{1.5-\gamma} - 1$ or equivalently

$$\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}(1+r_2) - 1 \,.$$

The discount rate for all other years in the present value calculation is r_2 and the present value of the after tax cash flows, corresponding to revenue payments at the updated CRF rate, is

(2.25)

$$(1-\mu)[c_2 K_0(1-s_2) + \delta_1 K_0 s_2] + \left(\frac{1+r_2}{1+r_1}\right)^{\gamma-.5}\left(\frac{1}{1+r_2}\right)\sum_{j=2}^{N}\frac{c_2 K_0(1-s_2) + \delta_j K_0 s_2}{(1+r_2)^{j-2}} \,.$$

Expression (2.25) is identical to (2.13) with $q = 1$, the present value of the after cash flows in Case II-B. This implies (2.14) with $q = 1$ is an valid restatement of (2.25) and an expression for c_2 can be obtained from (2.15) with $q = 1$,

(2.26)

$$c_2 = \frac{r_2(1+r_2)^{N-1}}{(1-s_2)\left\{(1+r_2)^N - 1 - \mu\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} r_2(1+r_2)^{N-1} + \left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right] r_2(1+r_2)^{N-1}\right\}}$$
$$\cdot\left\{\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\frac{K_1^{(1)}}{K_0} - \delta_1 s_2\left[\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5} - 1\right] + \mu\delta_1 s_2\left(\frac{1+r_1}{1+r_2}\right)^{\gamma-.5}\right.$$
$$\left. - s_2\sum_{j=1}^{N}\frac{\delta_j}{(1+r_2)^{j-1}}\right\}.$$

The expression for c_2 in (2.1) is obtained by replacing $K_1^{(1)}$ in (2.26) with the expression in (2.24).

This completes the derivation of (2.1).

3 Appendix - Useful Formulas

- A formula for the sum of a finite geometric series is given by

(3.1)

$$\sum_{j=H}^{W} v^j = \frac{v^H}{1-v}(1 - v^{W-H+1}) \,.$$

H and W are positive integers and v is any number except one ($v \neq 1$). Equation (3.1) is validated by noting that if S is the sum on the left hand side of (3.1), then $S - vS = v^H - v^{W+1}$ and solving for S gives the right hand side of (3.1).

- Formulas for the debt payment and the interest portion of the debt payment, assuming uniform annual end of year payments, are given by

(3.2)

$$P = D\frac{r_d(1+r_d)^N}{(1+r_d)^N - 1}$$

$$I_j = Dr_d(1+r_d)^{j-1}\left(\frac{(1+r_d)^{N-j+1} - 1}{(1+r_d)^N - 1}\right), \quad j = 1, \cdots, N$$

where D is the amount of the loan, r_d is the interest rate and N is the term of the loan. The formula for P follows by noting that the present value of the debt payments must equal the debt principal,

$$D = \sum_{j=1}^{N} \frac{P}{(1+r_d)^j} \,.$$

The right hand side is a finite geometric series and using (3.1) can we restated as

$$D = P\frac{1}{r_d}\left[\frac{(1+r_d)^N - 1}{(1+r_d)^N}\right].$$

Solving for P yields the formula in (3.2).

The interest payment for the year j is the product of the debt interest rate and the outstanding debt principal at the end of year $j-1$. The outstanding principal at the end of year 1 is the original debt less the repayment of the principal at the end of year 1 or

$$D_1 = D_0 - (P - I_1)$$

where D_0 is the original debt (i.e. $D_0 = D$), P is the debt payment and I_1 is the interest portion of the debt payment. Noting that $I_1 = r_d D_0$, D_1 is restated as

(3.3)

$$D_1 = D_0(1 + r_d) - P\,.$$

The outstanding debt at the end of year 2 is

$$D_2 = D_1 - (P - I_2)$$

and $I_2 = r_d D_1$ so that

$$D_2 = D_1(1 + r_d) - P\,.$$

Then replacing D_1 using (3.3),

$$D_2 = D_0(1 + r_d)^2 - P[1 + (1 + r_d)]\,.$$

This pattern continues and the outstanding principal at the end of year $j-1$ can be written as

$$D_{j-1} = D_0(1 + r_d)^{j-1} - P\sum_{k=1}^{j-1}(1 + r_d)^{k-1}\,.$$

Rewriting the sum as

$$\frac{1}{1 + r_d}\sum_{k=1}^{j-1}(1 + r_d)^k$$

and using (3.1) results in

$$D_{j-1} = D_0(1 + r_d)^{j-1} - P\frac{(1 + r_d)^{j-1} - 1}{r_d}\,.$$

Then replacing P and simplifying reveals

$$D_{j-1} = D_0(1+r_d)^{j-1}\left(\frac{(1+r_d)^{N-j+1}-1}{(1+r_d)^N-1}\right).$$

The formula for I_j in (3.2) is the product of D_{j-1} and r_d.

- A formula for a sum of discounted or compounded interest payments, assuming uniform end of year payments, is given by

(3.4)

$$\sum_{j=H}^{W} I_j v^j = \frac{Dr_d}{(1+r_d)^N-1}\left\{(1+r_d)^N\left(\frac{v^H}{1-v}\right)(1-v^{W-H+1}) - \frac{1}{1+r_d}\left(\frac{[(1+r_d)v]^H}{1-(1+r_d)v}\right)(1-[(1+r_d)v]^{W-H+1})\right\}$$

where H and W are positive integers and v is any number except one ($v \neq 1$).[25] Equation (3.4) follows by replacing I_j using (3.2) and writing

$$\sum_{j=H}^{W} I_j v^j = \frac{Dr_d}{(1+r_d)^N-1}\left\{(1+r_d)^N\sum_{j=H}^{W} v^j - \frac{1}{1+r_d}\sum_{j=H}^{W}[(1+r_d)v]^j\right\}$$

and then using (3.1) to rewrite the summations.

- Formulas for the debt payment and interest portion of the debt payment, assuming the half year convention and uniform annual payments, are given by

(3.5)

$$P = D\frac{r_d(1+r_d)^{N-1/2}}{(1+r_d)^N-1}$$

$$I_1 = D\left(\sqrt{1+r_d}-1\right)$$

$$I_j = D\frac{r_d(1+r_d)^{j-1}}{\sqrt{1+r_d}}\left(\frac{(1+r_d)^{N-j+1}-1}{(1+r_d)^N-1}\right), \quad j = 2,\cdots,N$$

25 The term v is typically a discount factor such as $v = 1/(1+r)$ or a compounding factor, $v = 1+r$, but the formula holds for any value of v except one ($v \neq 1$).

where D is the amount of the loan, r_d is the interest rate and N is the term of the loan.[26] The year 1 interest payment reflects a half year of interest.

- A formula for a sum of discounted or compounded interest payments, for $H > 1$ and assuming the half year convention, is given by

(3.6)

$$\sum_{j=H}^{W} I_j v^j = \frac{Dr_d(1+r_d)^{-3/2}}{(1+r_d)^N - 1}\left\{(1+r_d)^{N+1}\left(\frac{v^H}{1-v}[1-v^{W-H+1}]\right) - \frac{[(1+r_d)v]^H}{1-(1+r_d)v}\left(1-[(1+r_d)v]^{W-H+1}\right)\right\}.$$

Equation (3.6) follows by replacing I_j using (3.5) and writing

$$\sum_{j=H}^{W} I_j v^j = \frac{Dr_d(1+r_d)^{-3/2}}{(1+r_d)^N - 1}\left\{(1+r_d)^{N+1}\sum_{j=H}^{W} v^j - \sum_{j=H}^{W}[(1+r_d)v]^j\right\}.$$

Using (3.1) to replace the summations, the right hand side of (3.6) is obtained.

- The following relationship holds:

(3.7)

$$\frac{(1+\mathrm{r_e})^W - 1}{r_e} = (1+\mathrm{r_e})^{W-1} + \frac{(1+\mathrm{r_e})^{W-1} - 1}{r_e}$$

where W is a positive integer, $r_e \neq -1$ and $r_e \neq 0$. This follows by noting that

$$\sum_{j=1}^{W}\left(\frac{1}{1+\mathrm{r_e}}\right)^j = \frac{1}{1+\mathrm{r_e}} + \sum_{j=2}^{W}\left(\frac{1}{1+\mathrm{r_e}}\right)^j$$

and then using (3.1) to replace the summations.

- The following relationship holds:

26 The derivations are similar to the derivations for the end of year payment formulas in (3.2).

(3.8)

$$\frac{(1+\mathrm{r_e})^W - (1+\mathrm{r}_d)^W}{r_e - r_d} = (1+\mathrm{r_e})^{W-1} + (1+r_d)\frac{(1+\mathrm{r_e})^{W-1} - (1+\mathrm{r_d})^{W-1}}{r_e - r_d}.$$

where W is a positive integer, $r_e \neq -1$ and $r_e \neq r_d$. This follows by noting that

$$\sum_{j=1}^{W}\left(\frac{1+r_d}{1+\mathrm{r_e}}\right)^j = \frac{1+r_d}{1+\mathrm{r_e}} + \sum_{j=2}^{W}\left(\frac{1+r_d}{1+\mathrm{r_e}}\right)^j.$$

and then using (3.1) to replace the summations.