



The Commission may approve a contested offer of settlement only based on its merits.<sup>4</sup> A contested settlement may be approved on its merits under one of the four approaches set forth in *Trailblazer Pipeline Company*.<sup>5</sup> None of the approaches under *Trailblazer Pipeline Company* can be relied on for approval of the Offer. The Offer does not resolve the issues raised in the order setting this matter for hearing.<sup>6</sup> The order does not establish a just and reasonable basis for calculating a rate for an asynchronous solar facility.<sup>7</sup> There is no record supporting the revenue requirement as just and reasonable, including as a “package.” The Market Monitor represents the public interest in efficient and competitive markets. The settlement cannot be analyzed under the fair and reasonable standard applicable to uncontested settlements because the public interest in efficient and competitive markets is a central issue in this proceeding. There is no possibility of severing the issues in the manner contemplated under the *Trailblazer Pipeline Company* approaches.

Although the Commission encourages settlements, that policy is not a license to resolve cases at all costs.<sup>8</sup> An offer of settlement, as in this case, that is unfair, unreasonable,

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<sup>4</sup> 18 CFR § 385.602(h)(1) (“If the Commission determines that any offer of settlement is contested in whole or in part, by any party, the Commission may decide the merits of the contested settlement issues, if the record contains substantial evidence upon which to base a reasoned decision or the Commission determines there is no genuine issue of material fact.”)

<sup>5</sup> The four approaches for approving a settlement under *Trailblazer Pipeline Company* include: (i) addressing the contentions of the contesting party on the merits when there is any adequate record; (ii) approving a contested settlement as a package on the ground that the overall result of the settlement is just and reasonable; (iii) determining that the contesting party’s interest is sufficiently attenuated such that the settlement can be analyzed under the fair and reasonable standard applicable to uncontested settlements when the settlement benefits the directly affected settling parties; or (iv) preserving the settlement for the consenting parties while allowing contesting parties to obtain a litigated result on the merits. See *Trailblazer Pipeline Company*, 85 FERC ¶ 61,345 (1998)..

<sup>6</sup> *Bluestone Farm Solar, LLC*, 175 FERC ¶ 61,221 at P 13 (2021).

<sup>7</sup> Whether the AEP Method applies to asynchronous solar facilities is a question under active review in Commission proceedings. See NOI at PP 20–28.

<sup>8</sup> See, e.g., *Arkla Energy Resources*, 49 FERC ¶ 61,051, 61,217 (1989); *Transwestern Pipeline Co.*, 9 FERC ¶ 61,075, at 61,166 (1979).

or against the public interest must be rejected.<sup>9</sup> Instead, this case should proceed to hearing so that the record can be developed and issues of material fact and law can be resolved on the merits.

Article 3.5 of the Offer's proposed settlement provides: "[T]his Settlement shall not set precedent or otherwise provide guidance for any other generating project, solar or otherwise." If the Offer is approved, it will unavoidably indicate that solar facilities like Bluestone can receive compensation for reactive capability under Schedule 2 based on a filing using the *AEP* Method, and it would further establish a benchmark rate level for storage facilities. The public interest is better served by resolution of the issues raised in this proceeding on the basis of a full evidentiary record and reasoned analysis.

In the attached affidavit of Dr. Joseph E. Bowring ("Affidavit"), included pursuant to Rule 602(f)(4), Dr. Bowring explains why the *AEP* Method does not apply to solar facilities and why the requested revenue requirement is excessive.<sup>10</sup>

The issues raised in this proceeding have significant cost implications going forward. Failing to resolve these issues risks making payments to Bluestone and similar facilities for which they are not eligible. Resolution of these issues should not be deferred. There is significantly greater administrative efficiency if new issues are resolved now, rather than after years of baseless and arbitrary settlements.

In the Affidavit, Dr. Bowring explains why the level of the annual revenue requirement is excessive. The issue of an appropriate rate level under Schedule 2 needs resolution on the merits in this case and for future cases. The Market Monitor opposes the Offer. The Offer should be rejected. Further, settlement discussions in the proceeding should be terminated, and the issues raised in this proceeding should be decided on the merits.

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<sup>9</sup> 496 F.3d at 701.

<sup>10</sup> 18 CFR § 385.602(f)(4).

Joseph E. Bowring  
Independent Market Monitor for PJM  
President  
Monitoring Analytics, LLC  
2621 Van Buren Avenue, Suite 160  
Eagleview, Pennsylvania 19403  
(610) 271-8051  
*joseph.bowring@monitoringanalytics.com*

Respectfully submitted,



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Jeffrey W. Mayes

General Counsel  
Monitoring Analytics, LLC  
2621 Van Buren Avenue, Suite 160  
Eagleview, Pennsylvania 19403  
(610) 271-8053  
*jeffrey.mayes@monitoringanalytics.com*

Dated: April 28, 2022

## CERTIFICATE OF SERVICE

I hereby certify that I have this day served the foregoing document upon each person designated on the official service list compiled by the Secretary in this proceeding.

Dated at Eagleville, Pennsylvania,  
this 28<sup>th</sup> day of April, 2022.



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Jeffrey W. Mayes  
General Counsel  
Monitoring Analytics, LLC  
2621 Van Buren Avenue, Suite 160  
Eagleville, Pennsylvania 19403  
(610) 271-8053  
*jeffrey.mayes@monitoringanalytics.com*

**Attachment**  
**Exhibit Nos. IMM-0001–0003**

**Bowring Affidavit**  
**and Supporting Exhibits**



1 The proposed ARR is significantly higher than the average rate paid for reactive  
2 power in PJM. The proposed ARR exceeds the \$2,199 per MW-year level of the  
3 reactive revenue offset included in the PJM capacity market demand curve. The  
4 ARR should be capped at \$2,199 per MW-Year, or \$6.02 per MW-day. Even within  
5 the framework of Bluestone’s filing, the proposed annual carrying charge is  
6 incorrect and not adequately supported. The Market Monitor has calculated an  
7 appropriate capital recovery factor (“CRF”).

8 **Q 3. HAVE YOU PROVIDED TESTIMONY ON COMPENSATION FOR**  
9 **REACTIVE POWER IN OTHER PROCEEDINGS BEFORE THE FERC?**

10 A. Yes. I provided testimony in the *Panda Stonewall* reactive supply capability case  
11 (Docket No. ER21-1821-002), the *Whitetail Solar 3, et al.* reactive supply capability  
12 case (Docket No. ER20-1851-004 et al.), and the *Mechanicsville Solar, LLC*,  
13 reactive capability case (Docket No. ER21-2091). I provided an affidavit in support  
14 of opposition to an offer of settlement in the Meyersdale Storage, LLC, reactive  
15 supply capability case (ER21-864-000).

16 **Q 4. HAVE YOU PARTICIPATED IN OTHER FERC PROCEEDINGS**  
17 **RELATED TO REACTIVE POWER?**

18 A. Yes, I was invited to participate in a Commission technical conference and provided  
19 comments to the Commission in a proceeding convened to “discuss compensation  
20 for Reactive Supply and Voltage Control (Reactive Supply) within the Regional  
21 Transmission Organizations (RTOs) and Independent System Operators (ISOs).”<sup>2</sup>  
22 Specifically, the proceeding explored “types of costs incurred by generators for  
23 providing Reactive Supply capability and service; whether those costs are being  
24 recovered solely as compensation for Reactive Supply or whether recovery is also  
25 through compensation for other services; and different methods by which generators  
26 receive compensation for Reactive Supply (e.g., Commission-approved revenue  
27 requirements, market-wide rates, etc.).”<sup>3</sup>

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<sup>2</sup> *Reactive Supply Compensation in Markets Operated by Regional Transmission Organizations and Independent System Operators*, Docket No. AD16-17-000. I participated in a workshop convened June 20, 2016. The Market Monitor filed comments on July 29, 2016, and reply comments on September 20, 2016.

<sup>3</sup> *Id.* at 1.



1 On February 22 and March 23, 2022, the Market Monitor filed comments and reply  
2 comments responding to the Commission’s Notice of Inquiry in Docket No. AD22-  
3 2. The Notice of Inquiry included questions (at P 28 (question no. 5.d)) specifically  
4 addressing the over recovery issue. The Notice of Inquiry also included questions (at  
5 PP 20–28 (question no. 5) addressing the appropriateness of continuing to use the  
6 *AEP* Method in reactive capability proceedings, particularly proceedings to establish  
7 ARR for asynchronous generators.

8 The Market Monitor has intervened in and actively participated in FERC reactive  
9 power cases during the past five years.

10 The Market Monitor includes analysis and recommendations related to reactive  
11 power in the State of the Market Reports for PJM.<sup>4</sup>

12 **I.**

13 **Q 5. WHY SHOULD THE PROPOSED ANNUAL REVENUE REQUIREMENT**  
14 **BE REJECTED?**

- 15 A. The proposed black box payment to the 49.9 MW Bluestone Facility (ARR) of  
16 \$224,550.00 per year, or \$4,500 per MW-Year, or \$12.33 per MW-Day is excessive.

17 The *AEP* Method that is typically used in reactive capability proceedings was  
18 developed for use with generating facilities that have very different engineering and  
19 operational characteristics.<sup>5</sup> Regardless of whether the *AEP* Method is itself  
20 appropriate for use in establishing reactive capability costs, there is no  
21 corresponding method for defining the reactive capability costs, if any, associated  
22 with solar facilities.

23 Even by the standards of the *AEP* Method, an ARR of \$224,550.00 per year, or  
24 \$4,500 per MW-Year, or \$12.33 per MW-Day, is excessive, has not been

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<sup>4</sup> See, for example, *2021 State of the Market Report for PJM*, Section 10 (Ancillary Services Markets), which can be accessed at:  
<[http://www.monitoringanalytics.com/reports/PJM\\_State\\_of\\_the\\_Market/2021.shtml](http://www.monitoringanalytics.com/reports/PJM_State_of_the_Market/2021.shtml)>.

<sup>5</sup> See *American Electric Power Service Corp.*, 80 FERC ¶ 63,006 (1997), *aff'd*, 88 FERC ¶ 61,141 (1999); see also *Reactive Power Capability Compensation*, Notice of Inquiry, 177 FERC ¶ 61,118 (2021) (“Notice of Inquiry”).

1 demonstrated to have a rational basis, has not been demonstrated to be just and  
2 reasonable, and should be rejected. The average revenue requirement for reactive  
3 capability in PJM is about \$2,000 per MW-year. The revenue requirement for  
4 reactive capability included in the PJM Capacity Market is \$2,199 per MW-year.

5 There is no reasonable basis for such a wide disparity in cost for the same service.  
6 This result has not been explained or supported by Bluestone in its filing or its black  
7 box Offer. This disparity is inconsistent with competitive markets.

8 **II.**

9 **Q 6. HOW DO PJM MARKET RULES PROVIDE THE OPPORTUNITY TO**  
10 **RECOVER REACTIVE CAPABILITY COSTS?**

11 A. The PJM market rules that account for recovery of reactive revenues are built into  
12 the auction parameters, specifically, the VRR Curve. The PJM market rules  
13 explicitly account for recovery of reactive revenues of \$2,199 per MW-year through  
14 inclusion in the Net CONE parameter of the capacity market demand (VRR) curve.<sup>6</sup>  
15 The Net CONE parameter directly affects clearing prices by affecting both the  
16 maximum capacity price and the location of the downward sloping part of the VRR  
17 curve.

18 **Q 7. HOW DOES THE \$2,199 PER MW-YEAR NUMBER AFFECT THE**  
19 **DEMAND CURVE FOR CAPACITY?**

20 A. Elimination of the ancillary services revenue offset of \$2,199 per MW-Year would  
21 mean that the prices on the capacity market demand curve (VRR curve) for each  
22 MW level would be higher and the clearing prices for capacity that result from the  
23 interaction of the supply curve and the VRR curve, would be higher. The result  
24 would be the recovery of additional reactive capacity revenues in the price of  
25 capacity for all resources.

26 **Q 8. WHY IS THE DEMAND CURVE RELEVANT?**

27 A. If there were no nonmarket recovery of reactive revenue, there would be no reactive  
28 revenue offset to Net CONE and the demand curve would result in higher capacity  
29 market prices, all else held constant. If there were no nonmarket recovery of reactive

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<sup>6</sup> See OATT Attachment DD § 5.10(a)(v)(A).

1 revenue, the shape and location of the demand curve would give unit owners the  
2 opportunity to recover all reactive capability costs in the capacity market.

3 This is how the capacity market works for all the other costs of a generating plant  
4 other than short run marginal costs.

5 Payments based on cost of service approaches result in distortionary impacts on  
6 PJM markets. Elimination of the reactive revenue requirement and the recognition  
7 that capital costs are not distinguishable by function would increase prices in the  
8 capacity market. The VRR curve would shift to the right, the maximum VRR price  
9 would increase and offer caps in the capacity market would increase. The simplest  
10 way to address this distortion would be to recognize that all capacity costs are  
11 recoverable in the PJM markets.

12 The best approach would be to eliminate cost of service rates for reactive capability  
13 and allow for recovery of capacity costs through existing markets, including a  
14 removal of any offset for reactive revenue in offers and in the capacity market  
15 demand (VRR) curve. A second best approach would be to limit the revenue  
16 requirement that could be filed for under the OATT Schedule 2 to a level less than  
17 or equal to the reactive revenue credit included in the capacity market design, in the  
18 VRR curve Net CONE value, currently \$2,199 per MW-year.

19 **III.**

20 **Q 9. SHOULD THE AEP METHOD BE USED TO CALCULATE THE RATE**  
21 **FOR THE FACILITY?**

- 22 A. No. The current process does not actually compensate resources based on their costs  
23 of investment in reactive power capability. The *AEP* Method assigns costs between  
24 real and reactive power based on a unit's power factor. This is effectively an  
25 allocation based on a subjective judgment rather than actual investment. There are  
26 few if any identifiable costs incurred by generators in order to provide reactive  
27 power. Separately compensating resources based on a judgment based allocation of  
28 total capital costs was never and is not now appropriate in the PJM markets.  
29 Generating units are fully integrated power plants that produce both the real and  
30 reactive power required for grid operation.

31 The *AEP* Method originated with a regulated utility assigning costs between two  
32 sources of regulated revenue requirement. The practice persists in PJM only because

1 it provides a significant, guaranteed stream of riskless revenue. Generation owners  
2 have an incentive to maximize such guaranteed revenue streams.

3 There is no logical reason to have a separate fixed payment for any part of the  
4 capacity costs of generating units in PJM. If separate cost of service rates for  
5 reactive continue, they need to be correctly integrated in the PJM market design.

6 The best and straightforward solution is to remove cost of service rates for reactive  
7 supply capability and to remove the offset. Investment in generation can and should  
8 be compensated entirely through markets. Removing cost of service rules would  
9 avoid the significant waste of resources incurred to develop unneeded cost of  
10 service rates.

11 The result would be to pay generators market based rates for both real and reactive  
12 capacity.

13 The *AEP* Method never accurately reflected the investment costs of providing  
14 reactive power, nor was it intended to do so. The *AEP* Method is a cost of service  
15 allocation approach designed to assign the regulated revenue requirement for  
16 generating units to a regulated generation function and a regulated transmission  
17 function. The *AEP* Method was designed to split that cost recovery for generating  
18 units in a reasonable way, based on a judgment about what is reasonable. The *AEP*  
19 Method was never about actually identifying specific capital costs associated solely  
20 with the provision of reactive power. Cost of service approaches apply allocation  
21 factors to accounting line items based on assumptions. The assumptions are that X  
22 percent of a type of equipment at a generating plant is associated with reactive  
23 power while (1-X) percent is associated with real power. The false precision of the  
24 *AEP* Method is entirely based on arbitrary assumptions. Even proponents of the  
25 *AEP* Method do not assert that the goal is to recover only the costs associated with a  
26 specific portion of a power plant required for the production of reactive power, or,  
27 in most cases, that such identification is even possible. That is not what the *AEP*  
28 Method was intended to do or is intended to do. The *AEP* Method does not define  
29 costs that are uniquely associated with the production of reactive power.

30 The *AEP* Method is based on the incorrect premise that the capacity costs of an  
31 integrated power plant are separable. The capacity costs of an integrated power plant  
32 are not separable.

1 The fundamental flaw in the *AEP* Method approach is the assumption that the costs  
2 of providing reactive power are a function of the power factor. The power factor is  
3 the ratio of real power (expressed as megawatts or MW) to the total output (apparent  
4 power) of a generator (expressed as megavolt-amperes or MVA). The remaining  
5 output is reactive power (expressed as megavolt amperes reactive or MVAR). The  
6 allocator typically used by proponents of the *AEP* Method to assign costs to reactive  
7 power generation is  $(1 - (\text{PowerFactor})^2)$ . The power factor has superficial attraction  
8 as an appropriate allocator. The power factor is the core determinant of the reactive  
9 allocation factor in the *AEP* Method. Small changes in the power factor have large  
10 impacts on the costs allocated to reactive power. For a power factor of .95, the  
11 allocator is 9.75 percent while for a power factor of .90, the allocator is 19.00  
12 percent, and for a power factor of .70, the allocator is 51.00 percent. For a resource  
13 claiming a power factor of .70, does that mean that more than half of the generator's  
14 costs were incurred in order to provide reactive power? Does this mean that 51  
15 percent of the costs of the generator, exciter, and electrical equipment should be  
16 recovered through a cost of service rate? The answer to both questions is no. But  
17 resources have filed for guaranteed reactive revenue requirements on that basis.

18 The power factor has taken on somewhat mythical significance in the discussion of  
19 reactive power. There are frequently long discussions of power factors in reactive  
20 cases. The ratio of real to reactive power can vary significantly. The typical actual  
21 operating power factor of generators in PJM is determined by their voltage schedule  
22 and is usually between .97 and .99. The resultant *AEP* Method power factor  
23 allocator consistent with this actual reactive output of PJM generators and the actual  
24 tariff defined reactive output to generators is 5.91 to 1.99 percent. The nameplate  
25 power factor of thermal generating units is typically .85. But the nameplate power  
26 factor stamped on the generator at the factory and not based on actual operation on  
27 an actual grid. The nameplate power factor is meaningless for the actual operation  
28 of the power plant. The nameplate power factor does not mean that 27.75 percent of  
29 the power plant capital costs are associated with reactive power, although many  
30 resources have made that request because that is the power factor allocator based on  
31 the nameplate rating.

32 The power factor is not an appropriate allocator and does not reflect the actual  
33 capital costs associated with producing reactive power. The power factor has taken  
34 on a disproportionate significance in reactive rate cases because it is the single most

1 important allocator in the *AEP* Method. That significance illustrates the fundamental  
2 flaws in the *AEP* Method.

3 The power factor does not measure reactive capability. The power factor does not  
4 determine a plant's reactive capability. The power factor does not identify costs  
5 associated with reactive capability or provide a reasonable basis for allocating those  
6 costs to reactive or real power production.

7 **IV.**

8 **Q 10. WHAT ARE THE ISSUES WITH THE COMPANY'S PROPOSED**  
9 **ANNUAL CARRYING CHARGE CALCULATION?**

10 A. In its filing, Bluestone calculates an annual carrying charge which is a form of  
11 capital recovery factor (CRF). This CRF was initially presented in the prepared  
12 direct testimony of Donald J. Clayton on April 8, 2021.<sup>7</sup> Witness Clayton derived a  
13 fixed charge carrying rate which is the sum of a CRF and a fixed operating expense  
14 rate. The CRF presented by Witness Clayton is the sum of a sinking fund  
15 depreciation factor, and income tax factor, an offset for ADIT and the before tax  
16 weighted average cost of capital. The derivation does not accurately reflect the tax  
17 liability and the return on and the return of the capital investment.

18 The CRF is a rate, multiplied by the relevant investment, which defines the annual  
19 payment needed to provide a return on and of capital for the investment over a  
20 defined time period. CRFs include as inputs the weighted average cost of capital and  
21 its components, including the rate of return on equity and the interest rate on debt  
22 and the capital structure, in addition to depreciation and taxes. The Market  
23 Monitor's CRF accurately reflects the tax liability associated with the annual  
24 payment. The depreciation used in the calculation of the CRF should reflect the  
25 depreciation used for tax purposes. The sinking fund depreciation factor does not  
26 reflect the actual depreciation used by and therefore should not be used in the  
27 calculation of the revenue requirement for the facility.

28 Witness Clayton did not account for the actual tax treatment of the facility and did  
29 not adequately explain his tax treatment, did not account for the actual expected life

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<sup>7</sup> See BFS-1 at 18:15–21:5.

1 of the facility, did not adequately explain or support his depreciation method, and  
2 did not account for the actual cost of capital of the facility.

3 The total revenue requirement requested was based on the CRF. It is not possible to  
4 evaluate the details underlying the black box Offer.

5 **Q 11. HOW DO YOU PROPOSE TO CALCULATE THE CAPITAL RECOVERY**  
6 **FACTOR (CRF)?**

7 A. The best approach for calculating capital recovery over a defined period is the  
8 Capital Recovery Factor (CRF) used by the Market Monitor. I have attached to my  
9 affidavit as Exhibit No. IMM-0003, a Capital Recovery Factors (CRF) Technical  
10 Reference prepared by the Market Monitor. The technical reference explains in  
11 detail the components for accurately and consistently calculating a CRF.

12 The CRF should be required for use in all cost-based ratemaking provisions used in  
13 PJM, which now include black start service rates and reactive capability rates.

14 The CRF as proposed by the Market Monitor provides the necessary and sufficient  
15 level of revenue to pay the annual tax liability and the return on and return of a  
16 defined capital investment. The CRF approach proposed by the Market Monitor is  
17 based on the weighted average cost of capital (WACC) capital budgeting method.  
18 Under the WACC approach, the after tax cash flow is discounted at the after tax  
19 WACC rate and the payback of the investment in each cost recovery year reflects  
20 the defined capital structure. This approach can be efficiently reduced to a single  
21 formula for the CRF. FERC accepted this approach for black start service and  
22 directed PJM to include the CRF formula in the PJM tariff.<sup>8</sup> Additional details on  
23 the derivation of the CRF formula and examples are available in the MMU's CRF  
24 Technical Reference.

25 **Q 12. DOES THIS CONCLUDE YOUR AFFIDAVIT?**

26 A. Yes.

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<sup>8</sup> See *PJM Interconnection, L.L.C.*, 176 FERC ¶ 61,080 at PP 43–44 (2021).

**Exhibit IMM-0002**  
**PJM OATT Schedule 2**



**Exhibit IMM-0003**  
**Capital Recovery Factors**  
**Technical Reference**

**UNITED STATES OF AMERICA  
BEFORE THE  
FEDERAL ENERGY REGULATORY COMMISSION**

Bluestone Farm Solar, LLC

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Docket No. ER21-1696-000

**DECLARATION**

JOSEPH E. BOWRING states that I prepared the affidavit to which this declaration is attached with the assistance of the staff of Monitoring Analytics, LLC, and that the statements contained therein are true and correct to the best of my knowledge and belief. Monitoring Analytics, LLC, is acting in its capacity as the Independent Market Monitor for PJM.

Pursuant to Rule 2005(b)(3) (18 CFR § 385.2005(b)(3), citing 28 U.S.C. § 1746), I further state under penalty of perjury that the foregoing is true and correct.

Executed on April 28, 2022.



Joseph E. Bowring

PJM OATT Schedule 2 - Reactive Supply  
and Voltage Control from Generation or  
Other Sources Service

**SCHEDULE 2**  
**Reactive Supply and Voltage Control from**  
**Generation or Other Sources Service**

In order to maintain transmission voltages on the Transmission Provider's transmission facilities within acceptable limits, generation facilities and non-generation resources capable of providing this service that are under the control of the control area operator are operated to produce (or absorb) reactive power. Thus, Reactive Supply and Voltage Control from Generation or Other Sources Service must be provided for each transaction on the Transmission Provider's transmission facilities. The amount of Reactive Supply and Voltage Control from Generation or Other Sources Service that must be supplied with respect to the Transmission Customer's transaction will be determined based on the reactive power support necessary to maintain transmission voltages within limits that are generally accepted in the region and consistently adhered to by the Transmission Provider.

Reactive Supply and Voltage Control from Generation or Other Sources Service is to be provided directly by the Transmission Provider. The Transmission Customer must purchase this service from the Transmission Provider.

In addition to the charges and payments set forth in this Tariff, Schedule 2, Market Sellers providing reactive services at the direction of the Office of the Interconnection shall be credited for such services, and Market Participants shall be charged for such services, as set forth in Tariff, Attachment K-Appendix, section 3.2.3B.

The Transmission Provider shall administer the purchases and sales of Reactive Supply. PJMSettlement shall be the Counterparty to (a) the purchases of Reactive Supply from owners of Generation or Other Sources and Market Sellers and (b) the sales of Reactive Supply to Transmission Customers and Market Participants.

**Charges**

Purchasers of Reactive Supply and Voltage Control from Generation or Other Sources Service shall be charged for such service in accordance with the following formulae.

Monthly Charge for a purchaser receiving Network Integration Transmission Service or Point-to-Point Transmission Service to serve Non-Zone Load = Allocation Factor \* Total Generation Owner or other source owner Monthly Revenue Requirement

Monthly Charge for a purchaser receiving Network Integration Transmission Service or Point-to-Point Transmission Service to serve Zone Load = Allocation Factor \* Zonal Generation Owner or other source owner Monthly Revenue Requirement \* Adjustment Factor

Where:

Purchaser serving Non-Zone Load is a Network Customer serving Non-Zone Network Load or serving Network Load in a zone with no revenue requirement for Reactive Supply and Voltage Control from Generation or Other Sources Service, or a Transmission Customer where the Point of Delivery is at the boundary of the PJM Region.

Zonal Generation Owner or other source owner Monthly Revenue Requirement is the sum of the monthly revenue requirements for each generator or other source located in a Zone, as such revenue requirements have been accepted or approved, upon application, by the Commission.

Total Generation Owner or other source owner Monthly Revenue Requirement is the sum of the Zonal Generation or other source owner Monthly Revenue Requirements for all Zones in the PJM Region.

Allocation Factor is the monthly transmission use of each Network Customer or Transmission Customer per Zone or Non-Zone, as applicable, on a megawatt basis divided by the total transmission use in the Zone or in the PJM Region, as applicable, on a megawatt basis.

For Network Customers, monthly transmission use on a megawatt basis is the sum of a Network Customer's daily values of DCPZ or DCPNZ (as those terms are defined in Tariff, Part III, section 34.1) as applicable, for all days of the month.

For Transmission Customers, monthly transmission use on a megawatt basis is the sum of the Transmission Customer's hourly amounts of Reserved Capacity for each day of the month (not curtailed by PJM) divided by the number of hours in the day.

Adjustment Factor is determined as the sum of the total monthly transmission use in the PJM Region, exclusive of such use by Transmission Customers serving Non-Zone Load, divided by the total monthly transmission use in the PJM Region on a megawatt basis.

In the event that a single customer is serving load in more than one Zone, or serving Non-Zone Load as well as load in one or more Zones, or is both a Network Customer and a Transmission Customer, the Monthly Charge for such a customer shall be the sum of the Monthly Charges determined by applying the appropriate formulae set forth in this Schedule 2 for each category of service.

### **Payment to Generation or Other Source Owners**

Each month, the Transmission Provider shall pay each Generation Owner or other source owner an amount equal to the Generation Owner's or other source owner's monthly revenue requirement as accepted or approved by the Commission. In the event a Generation Owner or

other source owner sells a generator or other source which is included in its current effective monthly revenue requirement accepted or approved by the Commission, payments in that Generation Owner's or other source owner's Zone may be allocated as agreed to by the owners of the generator or other source in that Zone. Such Generation Owner or other source owners shall inform the Transmission Provider of any such agreement and submit either a filing to revise its cost-based rate or an informational filing in accordance with the requirements below in this Schedule 2. In the absence of agreement among such Generation Owners or other source owners, the Commission, upon application, shall establish the allocation. Generation Owners shall not be eligible for payment, pursuant to this Schedule 2, of monthly revenue requirement associated with those portions of generating units designated as Behind The Meter Generation. The Transmission Provider shall post on its website a list for each Zone of the annual revenue requirements for each Generation Owner receiving payment within such Zone and specify the total annual revenue requirement for all of the Transmission provider.

At least 90 days prior to the Deactivation Date or disposition date of a generator or other source receiving payment in accordance with a Commission accepted or approved revenue requirement for providing reactive supply and voltage control service under this Schedule 2, the Generation Owner or other source owner must either:

(1) submit to the Commission the appropriate filings to terminate or revise its cost-based revenue requirement for supplying reactive supply and voltage control service under this Schedule 2 to account for the deactivated or transferred generator or other source; or

(2) provide to the Transmission Provider and file with the Commission an informational filing that includes the following information:

- (i) the acquisition date, Deactivation Date, and transfer date of the generator or other source;
- (ii) an explanation of the basis for the decision by the Generation Owner or other source owner not to terminate or revise the cost-based rate approved or accepted by the Commission associated with the planned generator or other source deactivation or disposition;
- (iii) a list of all of the generators or other sources covered by the Generation Owner's or other source owner's cost-based tariff from the date the revenue requirement was first established until the date of the informational filing;
- (iv) the type (i.e., fuel type and prime mover) of each generator or other source;
- (v) the actual (site-rated) megavolt-ampere reactive ("MVAR") capability, megavolt-ampere ("MVA") capability, and megawatt capability of each generator or other source, as supported by test data; and
- (vi) the nameplate MVAR rating, nameplate MVA rating, nameplate megawatt rating, and nameplate power factor for each generator or other source.

The Generation Owner or other source owner must submit the informational filing in the docket in which its cost-based revenue requirement was approved or accepted by the Commission or as otherwise directed by the Commission.

The requirement to submit the filings at least 90 days prior to the Deactivation Date or disposition date of a generator or other source shall not apply to generators or other source deactivations or transfers occurring between June 18, 2015, and September 16, 2015. For generator or other source deactivations or transfers occurring between June 18, 2015, and September 16, 2015, the Generation Owner or other source owner shall submit the informational filing or filings to terminate or revise its cost-based revenue requirement by September 16, 2015.



# Capital Recovery Factors (CRF) Technical Reference

Monitoring Analytics, LLC

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## Table of Contents

1	The Basics of CRF .....	3
1.1	CRF That Reflect Taxable Income .....	4
1.2	Half Year Convention.....	7
1.3	Proof of Proposition 1.2.....	11

## 1 The Basics of CRF

A capital recovery factor (CRF) is used to convert the principal amount of a capital investment into an equivalent stream of uniform payments. A typical CRF formula found in engineering economics textbooks is given in equation (1.1).<sup>1</sup>

(1.1)

$$CRF = \frac{r(1+r)^N}{(1+r)^N - 1}$$

Variable  $r$  is an interest rate,  $N$  is the number of uniform annual payments and the payments are assumed to occur at the end of year. To derive equation (1.1) the CRF is first denoted by  $c$ , allowing the annual payment to be stated as  $A = cK$  where  $K$  is the capital investment. Then  $c$  is the value that solves the following present value equation,

$$\begin{aligned} K &= \sum_{j=1}^N \frac{cK}{(1+r)^j} \\ &= cK \sum_{j=1}^N \left(\frac{1}{1+r}\right)^j \end{aligned}$$

The summation in the equation above is a finite geometric series. A general formula for the sum of a finite geometric series is given by

(1.2)

$$\sum_{j=H}^W v^j = \frac{v^H}{1-v} (1 - v^{W-H+1}).$$

$H$  and  $W$  are positive integers and  $v$  is any number except one ( $v \neq 1$ ). It is straightforward exercise to show that equation (1.2) is valid.<sup>2</sup>

Using equation (1.2) with  $H = 1$ ,  $W = N$  and  $v = 1/(1+r)$  yields

$$\sum_{j=1}^N \left(\frac{1}{1+r}\right)^j = \frac{(1+r)^N - 1}{r(1+r)^N}.$$

Replacing the summation in the present value equation yields

$$K = cK \left( \frac{(1+r)^N - 1}{r(1+r)^N} \right)$$

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<sup>1</sup> For example, see pages 21-22 in "Economic Evaluation and Investment Decision Methods," Stermole, F.J. and Stermole, J.M. (1993).

<sup>2</sup> If  $S$  is the sum on the left hand side of equation (1.2), then  $S - vS = v^H - v^{W+1}$  and solving for  $S$  gives the right hand side of (1.2).

and solving for  $c$  produces equation (1.1).

## 1.1 CRF That Reflect Taxable Income

The revenue that results from a capital investment is taxable income. The revenue payment  $A$ , obtained by multiplying the capital investment amount  $K$  by the CRF in equation (1.1), would be too low in cases where the revenue is taxable. The goal, in the presence of taxes, is to have a CRF for which the product  $CRF \cdot K$  yields an annual payment  $A$  that will provide the necessary and sufficient level of revenue to cover the investors' annual tax payments, and the return on and return of the capital investment. In other words, over the life of the project, the revenue in excess of the tax payments and investment return should equal the original capital investment. The annual revenue payment can be determined by solving an equation where the present value of the after tax cash flows resulting from the annual revenue payment is equal to the initial capital investment.

The composition of the after tax cash flow is dependent upon the capital budgeting model. The weighted average cost of capital (WACC) approach was used to develop the CRF for PJM Black Start Service which was accepted by FERC in August 2021.<sup>3 4</sup> The WACC approach to capital budgeting discounts the after tax cash flow at the after tax weighted average cost of capital rate and payback of the investment in each recovery year reflects the assumed debt and equity financing structure.<sup>5</sup> The CRF must satisfy the following present value equation,

$$K = \sum_{j=1}^N \frac{CF_j}{(1+r)^j} .$$

$K$  is the capital investment,  $CF_j$  is the after tax cash flow for year  $j$ ,  $r$  is the WACC rate, and the revenue, tax and debt payments are assumed to occur at the end of the year. The model variables are defined in Table 1-1. In the WACC model, the after tax cash flow is revenue net of taxes, and the tax calculation includes an offset for depreciation. The after tax cash flow for year  $j$  is

$$\begin{aligned} CF_j &= cK - (cK - \delta_j K)s \\ &= cK(1 - s) + \delta_j Ks \end{aligned}$$

<sup>3</sup> 176 FERC ¶ 61,080 (August 10, 2021) at 43-44.

<sup>4</sup> Additional details on the weighted average cost of capital approach to capital budgeting can be found in Section 17.3 in "Corporate Finance," Ross, Westerfield, Jaffe, 4<sup>th</sup> Edition, 1996.

<sup>5</sup> The after tax weighted average cost of capital rate is equal to *Equity Funding Percent x Equity Rate + Debt Funding Percent x Debt Interest Rate x (1- Effective Tax Rate)*.

where  $c$  is the CRF,  $K$  is the total capital investment including debt and equity,  $cK$  is the annual revenue payment,  $s$  is the effective tax rate and  $\delta_j$  is the depreciation factor for year  $j$ . Upon replacing  $CF_j$  in the present value equation

$$K = cK(1 - s) \sum_{j=1}^N \frac{1}{(1 + r)^j} + Ks \sum_{j=1}^N \frac{\delta_j}{(1 + r)^j}.$$

Equation (1.2) with  $H = 1$ ,  $W = N$  and  $v = 1/(1 + r)$  gives

$$\sum_{j=1}^N \frac{1}{(1 + r)^j} = \frac{(1 + r)^N - 1}{r(1 + r)^N}$$

and substituting into the previous equation results in

$$K = cK(1 - s) \left( \frac{(1 + r)^N - 1}{r(1 + r)^N} \right) + Ks \sum_{j=1}^N \frac{\delta_j}{(1 + r)^j}.$$

Solving for  $c$  yields the CRF formula in equation (1.3).

(1.3)

$$CRF = \frac{r(1 + r)^N}{(1 - s)[(1 + r)^N - 1]} \left\{ 1 - s \sum_{j=1}^N \frac{\delta_j}{(1 + r)^j} \right\}$$

**Table 1-1 Variable descriptions for the WACC capital budgeting model**

Variable	Description
$r$	After tax weighted average cost of capital
$s$	Effective tax rate
$N$	Cost recovery period
$\delta_j$	Depreciation factor for recovery year $j$

Substituting the parameter values shown in Table 1-2 into the CRF formula, assuming a five year capital recovery period and straight line depreciation yields a CRF of 0.274938. With a capital investment of \$1 million, the annual payment is \$274,938.

Table 1-3 provides a cash flow summary for a \$1 million capital investment with a five year cost recovery period that uses straight line depreciation. The revenue for each year, equal to the product of the CRF and the capital investment amount, is \$274,938. The tax payment for each year is equal to the effective tax rate times the revenue net of depreciation. The return on the capital investment in year 1 is equal to the product of the WACC rate and the initial capital investment of \$1,000,000.

**Table 1-2 Financial parameter and tax assumptions<sup>6</sup>**

Parameter	Parameter Value
Equity Funding Percent	50.0000%
Debt Funding Percent	50.0000%
Equity Rate	12.0000%
Debt Interest Rate	7.0000%
Federal Tax Rate	21.0000%
State Tax Rate	9.0000%
Effective Tax Rate (s)	28.1100%
After tax Weighted Average Cost of Capital (r)	8.5162%

After accounting for the tax payment and return on investment in year 1, \$168,711 is available as payback to the investors. The remaining capital investment is \$831,289 at the end of year 1. The year 2 return on investment is the product of the WACC rate and the remaining capital investment at the end of year 1. Payback to investors is \$183,079 in year 2. The cash flows for years 3 through 5 are analogous to the year 2 cash flow.

**Table 1-3 Cash flow summary for 5 year, \$1 million investment with straight line depreciation<sup>7</sup>**

Recovery Year	1	2	3	4	5
Revenue	\$274,938	\$274,938	\$274,938	\$274,938	\$274,938
Depreciation	\$200,000	\$200,000	\$200,000	\$200,000	\$200,000
Tax Payment	\$21,065	\$21,065	\$21,065	\$21,065	\$21,065
Return on capital investment	\$85,162	\$70,794	\$55,202	\$38,283	\$19,923
Capital investment payback	\$168,711	\$183,079	\$198,670	\$215,590	\$233,949
Remaining capital investment	\$831,289	\$648,209	\$449,539	\$233,949	\$0

After the final revenue payment in year 5, the remaining capital investment is reduced to \$0. Summing horizontally across the capital investment payback row in Table 1-3 produces \$1,000,000. This example illustrates that the revenue payment determined by the CRF provides the necessary and sufficient annual revenue to pay the taxes associated with the revenue payment as well as the required return on and return of the capital investment. This important point is established as a general result in the following proposition.

*Proposition 1.1.* The CRF given by equation (1.3) is the unique value, assuming a WACC capital budgeting model with end of year payments, for which the resulting annual revenue payment is

<sup>6</sup> The effective tax rate (parameter s in the formula) is equal to  $State\ Tax\ Rate + Federal\ Tax\ Rate \times (1 - State\ Tax\ Rate)$ .

<sup>7</sup> WACC model with end of year revenue and tax payments.

necessary and sufficient, over the term of the investment, to provide for the annual tax liability and the return on and return of the capital investment.

## 1.2 Half Year Convention

The revenue and tax payments would likely be made on a monthly or quarterly basis rather than occurring at the end of the year. A better model with respect to the timing of the revenue and tax payments is obtained by assuming the revenue and tax payments occur at the midpoint of each year. To derive a CRF corresponding to midyear revenue and tax payments, the present value equation from the previous section is modified to reflect the new timing assumption. Each after tax cash flow amount is assumed to occur a half year earlier than in the previous model. The revised present value equation is

$$K = \sum_{j=1}^N \frac{CF_j}{(1+r)^{j-0.5}},$$

or equivalently,

$$K = \sqrt{1+r} \sum_{j=1}^N \frac{CF_j}{(1+r)^j}.$$

Making the substitution,

$$CF_j = cK(1-s) + \delta_j Ks$$

and solving for  $c$  yields equation (1.4).

(1.4)

$$CRF = \frac{r(1+r)^N}{(1-s)[(1+r)^N - 1]} \left\{ \frac{1}{\sqrt{1+r}} - s \sum_{j=1}^N \frac{\delta_j}{(1+r)^j} \right\}$$

Using the parameter values in Table 1-2, with a five year capital cost recovery period and straight line depreciation, equation (1.4) yields a CRF of 0.260798. With an initial capital investment of \$1 million, the annual payment is \$260,798. Table 1-4 shows the corresponding cash flow summary.

**Table 1-4 Cash flow summary for 5 year, \$1 million investment with half year convention**

Service Year	1	2	3	4	5
Revenue	\$260,798	\$260,798	\$260,798	\$260,798	\$260,798
Depreciation	\$200,000	\$200,000	\$200,000	\$200,000	\$200,000
Tax Payment	\$17,090	\$17,090	\$17,090	\$17,090	\$17,090
Return on Capital Investment	\$41,711	\$67,959	\$52,992	\$36,751	\$19,126
Payback of Capital Investment	\$201,997	\$175,749	\$190,716	\$206,957	\$224,582
Remaining Capital Investment	\$798,003	\$622,255	\$431,539	\$224,582	\$0

The calculation of the values in Table 1-4 is identical to the corresponding values in Table 1-3 except that the year 1 return on investment reflects a half year period. The return on investment in year 1 is equal to the product of the capital investment and the half year rate of return  $\sqrt{1+r} - 1$ . The cash flow summary shows that the revenue payment determined by the CRF is necessary and sufficient to pay the taxes associated with the revenue payment as well as the required return on and return of the capital investment.

Changing the depreciation assumption to 3 year MACRS produces a CRF of 0.254231. The MACRS depreciation factors are shown in Table 1-8. The lower CRF relative to the straight line depreciation example reflects the lower tax payment under MACRS due to the accelerated depreciation schedule. In years 1 and 2, the tax payment in Table 1-5 is negative due to the accelerated depreciation assumption.<sup>8</sup> The cash flow summary in Table 1-5 shows that the revenue payment determined by the CRF, using 3 year MACRS depreciation, is at the necessary and sufficient level to provide for the taxes associated with the revenue payment as well as the required return on and return of the capital investment.

**Table 1-5 Cash flow summary for 5 year, \$1 million investment with 3 year MACRS**

Service Year	1	2	3	4	5
Revenue	\$254,231	\$254,231	\$254,231	\$254,231	\$254,231
Depreciation	\$333,300	\$444,500	\$148,100	\$74,100	\$0
Tax Payment	(\$22,226)	(\$53,485)	\$29,833	\$50,635	\$71,464
Return on Capital Investment	\$41,711	\$65,170	\$44,515	\$29,195	\$14,343
Payback of Capital Investment	\$234,747	\$242,546	\$179,883	\$174,401	\$168,424
Remaining Capital Investment	\$765,253	\$522,708	\$342,825	\$168,424	\$0

The depreciation assumption has a significant impact on the CRF level. Generally, the faster the capital is depreciated for tax purposes, the lower the CRF. The Tax Cuts and Jobs Act (TCJA), signed into law on December 22, 2017 included bonus depreciation rates applicable to capital investments placed in service after September 27, 2017.<sup>9 10</sup> Capital investments placed into service after September 27, 2017 and before January 1, 2023, are eligible for 100 percent bonus depreciation.<sup>11</sup>

<sup>8</sup> It is assumed that the capital investor would use the negative tax liability from this project as an offset against the tax liability resulting from other revenue.

<sup>9</sup> Tax Cuts and Jobs Act, Pub. L. No. 115-97, 131 Stat. 2096, Stat. 2105 (2017).

<sup>10</sup> 26 U.S. Code §11(b)

<sup>11</sup> Bonus depreciation is 100 percent for capital investments placed in service after September 27, 2017 and before January 1, 2023. Bonus depreciation is 80 percent for capital investments placed in service after December 31, 2022 and before January 1, 2024, and the bonus depreciation level is reduced by 20

Assuming 100 percent bonus depreciation results in a CRF of 0.247523. The corresponding cash flow summary is given in Table 1-6. The CRF for straight line depreciation for a five year cost recovery period is 5.3 percent higher than the CRF corresponding to 100 percent bonus depreciation.

**Table 1-6 Cash flow summary for 5 year, \$1 million investment with bonus depreciation**

Service Year	1	2	3	4	5
Revenue	\$247,523	\$247,523	\$247,523	\$247,523	\$247,523
Depreciation	\$1,000,000	\$0	\$0	\$0	\$0
Tax Payment	(\$211,521)	\$69,579	\$69,579	\$69,579	\$69,579
Return on Capital Investment	\$41,711	\$49,621	\$38,692	\$26,834	\$13,965
Payback of Capital Investment	\$417,334	\$128,324	\$139,252	\$151,111	\$163,980
Remaining Capital Investment	\$582,666	\$454,343	\$315,091	\$163,980	\$0

The CRF for a capital investment with a 20 year recovery period is 0.103149 and the corresponding cash flow summary is given in Table 1-7 for a capital investment totaling \$10,000,000.

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percent for each subsequent year through 2026. Capital investments placed in service after December 31, 2026 are not eligible for bonus depreciation. See 26 U.S. Code §168(k)(6)(A).



**Table 1-7 Cash flow summary for 20 year, \$10 million investment with bonus depreciation**

Service Year	Revenue	Depreciation	Tax Payment	Return on Capital Investment	Payback of Capital Investment	Remaining Capital Investment
1	\$1,031,492	\$10,000,000	(\$2,521,048)	\$417,109	\$3,135,431	\$6,864,569
2	\$1,031,492	\$0	\$289,952	\$584,597	\$156,943	\$6,707,626
3	\$1,031,492	\$0	\$289,952	\$571,231	\$170,308	\$6,537,318
4	\$1,031,492	\$0	\$289,952	\$556,728	\$184,812	\$6,352,506
5	\$1,031,492	\$0	\$289,952	\$540,989	\$200,551	\$6,151,955
6	\$1,031,492	\$0	\$289,952	\$523,910	\$217,630	\$5,934,325
7	\$1,031,492	\$0	\$289,952	\$505,376	\$236,164	\$5,698,161
8	\$1,031,492	\$0	\$289,952	\$485,264	\$256,276	\$5,441,886
9	\$1,031,492	\$0	\$289,952	\$463,439	\$278,101	\$5,163,785
10	\$1,031,492	\$0	\$289,952	\$439,756	\$301,784	\$4,862,001
11	\$1,031,492	\$0	\$289,952	\$414,055	\$327,484	\$4,534,517
12	\$1,031,492	\$0	\$289,952	\$386,166	\$355,373	\$4,179,143
13	\$1,031,492	\$0	\$289,952	\$355,902	\$385,638	\$3,793,505
14	\$1,031,492	\$0	\$289,952	\$323,061	\$418,479	\$3,375,026
15	\$1,031,492	\$0	\$289,952	\$287,422	\$454,117	\$2,920,909
16	\$1,031,492	\$0	\$289,952	\$248,749	\$492,791	\$2,428,118
17	\$1,031,492	\$0	\$289,952	\$206,782	\$534,758	\$1,893,361
18	\$1,031,492	\$0	\$289,952	\$161,241	\$580,298	\$1,313,062
19	\$1,031,492	\$0	\$289,952	\$111,822	\$629,717	\$683,345
20	\$1,031,492	\$0	\$289,952	\$58,195	\$683,345	\$0

In each example, the annual revenue payment, equal to the product of the capital investment and the CRF obtained from equation (1.4) is the necessary and sufficient revenue amount to cover the tax liability and the return on and return of the investment capital. This observation is generalized in the following proposition.

*Proposition 1.2.* The CRF given by equation (1.4) is the unique value, assuming a WACC capital budgeting model with the half year convention, for which the resulting annual revenue payment is necessary and sufficient, over the term of the investment, to pay the annual tax liability and the return on and return of the capital investment.

**Table 1-8 Modified Accelerated Cost Recovery System (MACRS) with half year convention<sup>12</sup>**

Year	3 year Depreciation Factors	5 year Depreciation Factors	10 year Depreciation Factors	15 year Depreciation Factors	20 year Depreciation Factors
1	33.33%	20.00%	10.00%	5.00%	3.750%
2	44.45%	32.00%	18.00%	9.50%	7.219%
3	14.81%	19.20%	14.40%	8.55%	6.677%
4	7.41%	11.52%	11.52%	7.70%	6.177%
5		11.52%	9.22%	6.93%	5.713%
6		5.76%	7.37%	6.23%	5.285%
7			6.55%	5.90%	4.888%
8			6.55%	5.90%	4.522%
9			6.56%	5.91%	4.462%
10			6.55%	5.90%	4.461%
11			3.28%	5.91%	4.462%
12				5.90%	4.461%
13				5.91%	4.462%
14				5.90%	4.461%
15				5.91%	4.462%
16				2.95%	4.461%
17					4.462%
18					4.461%
19					4.462%
20					4.461%
21					2.231%

### 1.3 Proof of Proposition 1.2

*Proposition 1.2.* The CRF given by equation (1.4) is the unique value, assuming a WACC capital budgeting model with the half year convention, for which the resulting annual revenue payment is necessary and sufficient, over the term of the investment, to pay the annual tax liability and the return on and return of the capital investment.

*Proof.*  $K_0$  is the initial capital invested and  $K_j, j \geq 1$ , represents the capital investment remaining at the midpoint of cost recovery year  $j$ .  $K_1$  is the remaining capital investment at the midpoint of year 1 after using the year 1 revenue net of taxes and return on investment, as a payback to investors. The proposition states that the CRF in equation (1.4) is the unique value that will result in  $K_N = 0$ . Representing the CRF in equation (1.4) as  $c$ , the year 1 revenue net of taxes and return on investment is

<sup>12</sup> See Appendix A, Table A-1, IRS Publication 946, United States Department of Treasury (2020).

$$cK_0(1-s) + \delta_1 K_0 s - K_0(\sqrt{1+r} - 1).$$

The rate of return on the investment reflects a half year of return due to the half year convention. The equity investment that remains at the midpoint of year 1 is

$$\begin{aligned} K_1 &= K_0 - \left( cK_0(1-s) + \delta_1 K_0 s - K_0(\sqrt{1+r} - 1) \right) \\ &= K_0 \sqrt{1+r} - cK_0(1-s) - \delta_1 K_0 s. \end{aligned}$$

The year 2 revenue net of taxes and return on investment is

$$cK_0(1-s) + \delta_2 K_0 s - rK_1$$

and the capital investment that remains at the midpoint of year 2 is

$$K_2 = K_1(1+r) - cK_0(1-s) - \delta_2 K_0 s.$$

Substitution for  $K_1$  yields

$$K_2 = K_0(1+r)^{3/2} - cK_0(1-s)[(1+r) + 1] - [\delta_1(1+r) + \delta_2]K_0 s.$$

Repeating this process through the end of the cost recovery period yields

(1.5)

$$K_N = K_0(1+r)^{N-1/2} - cK_0(1-s) \sum_{j=1}^N (1+r)^{j-1} - K_0 s \sum_{j=1}^N \delta_j (1+r)^{N-j}.$$

Equation (1.2) with  $H = 1$ ,  $W = N$  and  $v = 1+r$  gives

$$\sum_{j=1}^N (1+r)^{j-1} = \frac{1}{1+r} \sum_{j=1}^N (1+r)^j = \frac{(1+r)^N - 1}{r}.$$

Replacing the first summation in equation (1.5) yields

(1.6)

$$K_N = K_0(1+r)^{N-1/2} - cK_0(1-s) \left( \frac{(1+r)^N - 1}{r} \right) - K_0 s \sum_{j=1}^N \delta_j (1+r)^{N-j}.$$

Replacing  $c$  in (1.6) with the CRF formula in (1.4) results in  $K_N = 0$ . Equation (1.6) also establishes the uniqueness of the CRF. If there are two CRF values, for instance  $c_1$  and  $c_2$ , satisfying the proposition, then each will produce  $K_N = 0$  and one can quickly deduce from the equation (1.6) that  $c_1 = c_2$ .